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Abstract

Opportunistic schedulers such as MaxRate and Proportional Fair are known for trading off throughput and fairness of users in cellular networks. In this paper we show how to achieve maximum fairness without sacrificing throughput. We propose a novel solution that integrates opportunistic scheduling design principles and cooperative device-to-device communication capabilities in order to improve both fairness and capacity in cellular networks. We develop a mathematical approach and design a smart tie-breaking mechanism which enhances the fairness achieved by the MaxRate scheduler. We show that users that cooperatively form clusters benefit from both higher throughput and fairness. Our scheduling mechanism is simple to implement and scales linearly with the number of clusters, and is able to achieve equal or better fairness than Proportional Fair schedulers.

I. Introduction

The fourth generation (4G LTE-advanced) cellular technologies revolutionized 21^{th} century communications with advanced physical layer and improved medium access techniques. Nevertheless, exponential consumer traffic growth took over the competition from 4G cellular networks and forced the operators to seek alternatives to enhance the cellular capacity *effectively*. Cellular networks have several limitations such as computational constraints, fairness concerns and expensive infrastructure. Hence, most of the proposed solutions to sustain this traffic growth

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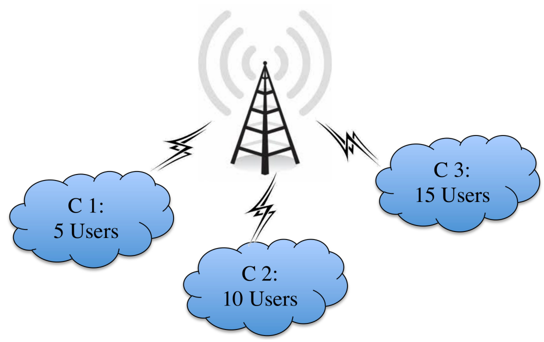


Fig. 1. Example scenario: three clusters in a base station, with five, ten and fifteen mobile users, respectively.

are disqualified for practical deployment due to high complexity, unfairness, high costs, and marginal enhancement that they exhibit. Given all the above limitations, an increasing number of researchers and practitioners suggest to allow and/or ask the mobile users to contribute in improving the network performance. Interestingly, within the past few years, many mobile users upgraded their conventional cell phones to 3G/4G smartphones with WiFi and bluetooth capabilities [1], and many other modern devices (e.g., tablets, laptops) now support multiple communications technologies. This multi-interface capability can be used to foster device-to-device (D2D) communications among mobile users [2], [3], without necessarily consuming cellular resources, e.g., by using license-free spectrum for short range communications (*outbound relay*).

It has been shown that the cellular throughput can be dramatically improved by using opportunistic schedulers such as MaxRate [4] and Proportional Fair [5]. The opportunistic schedulers proposed for cellular networks face a trade off between throughput and fairness when it comes to prioritizing the users based on their channel qualities [6], [7]. Therefore, with the existing cellular architectures, opportunistic schedulers cannot achieve maximum throughput and fairness at the same time, unless all mobile users experience the very same transmission channel [8], [9].

In contrast, in this manuscript, we show how to evolve the cellular architecture by leveraging D2D communications to achieve maximum throughput and fairness. In particular, we explore the possible gain from having D2D connections within *clusters* of mobile users, as shown in Fig. 1, where each cluster is treated by the base station as a regular mobile user in an LTE cell. We propose to change the normal LTE operation as follows: at each scheduling frame, the scheduled mobile user is responsible for the traffic of its *entire* cluster, i.e., it acts as *cluster head*. The LTE traffic managed by the cluster head is then immediately exchanged within the cluster via D2D communications on a secondary wireless interface (WiFi). Note that the cluster head is, in principle, *opportunistically* different at any frame, thereby achieving maximum throughput.

In turn, fairness is achieved in the following way. The schedulers select some of the connections for transmission, however, there are situations in which two or more users are in *tie*, i.e., they can be scheduled with the same modulation and coding scheme. These ties are usually ignored or broken randomly [10]. In contrast, we show that a smart *tie-breaking* strategy allows to compensate for the channel quality differences experienced by the data connections which are active in the cellular network. So far, nobody has investigated the possibility of enhancing fairness of opportunistic schedulers by utilizing tie-breaking methods. Thus, we are the first to explore tie-breaking mechanisms for improving fairness of MaxRate scheduler, though some other opportunistic schedulers could also be enhanced with our approach. We select the MaxRate scheduler since it maximizes system throughput, and we show that improved fairness levels can be achieved without paying any throughput cost, i.e., *we study how to break ties in order to improve connection fairness while maintaining the maximum cell throughput.*

Therefore, we propose a novel solution that integrates *opportunistic scheduling design principles* and *cooperative D2D communication capabilities* to improve both fairness and throughput in cellular networks. The main contributions of this paper can be summarized as follows:

1) we develop a mathematical framework for the evaluation of fairness in an architecture based on cooperative D2D communications and opportunistic scheduling in cellular networks;
2) we design and analyze novel tie-breaking mechanisms to provide improved fairness under MaxRate cluster scheduling;
3) we evaluate the performance of the proposed solution through an extensive numerical simulation study.

The remainder of this paper is organized as follows. In Section II, we review the literature on works relevant to our proposal. Our system model is presented in Section III. Section IV analytically tackles fairness issues which are due to MaxRate scheduling, and discusses a few mechanisms to improve the level of fairness in the system without reducing the total throughput. For the simple case of a system with two active data connections, we analytically design *MaxRate-MaxFair*, a mechanism that achieves the highest possible fairness and maximum throughput. Inspired by the results of the two-connection case, in Section V, we propose four heuristics for the generic case of N connections to be scheduled. In Section VI, we use numerical simulations to evaluate the throughput and the fairness that can be achieved with our proposals in a variety of realistic scenarios. Our schemes can achieve 50% higher throughput than conventional networks while providing a user fairness index close to one. Finally, we summarize and conclude the paper in Section VII.

II. Related Work

a) Device-to-Device (D2D) communications: D2D communications include all technologies that allow direct communications among users without involvement of an infrastructure or an access point [2]. D2D communication may be utilized in a variety of scenarios such as cooperative communications, packet forwarding, and relaying. In [2] and [3], authors explore some applications of D2D communications in cellular networks such as P2P, multiplayer gaming, and multicast transmissions. Doppler *et al.* [11] explore D2D communication establishment and management in LTE-advanced networks. They show that D2D communications using the very same cellular resources (*inbound relay*) increase the throughput by up to 65%. The authors of [12] propose to introduce *fixed ad hoc relay stations*, operating on licensed or unlicensed band, to enable P2P communications within a cell or adjacent cells. Their approach reduces the call blocking probabilities by routing new calls to less congested cells. The authors of [13], [14] propose to use D2D communications by forming clusters among mobile users with single antenna to emulate a MIMO device. Yu *et al.* [15] propose D2D communications in cellular networks for local traffic handling. D2D transmissions are meant to handle communications among two mobile devices, however users do not help each other to relay traffic to the base station. Also, all transmissions occur over the same interface as cellular communications, and D2D resources are allocated by the base station.

b) Opportunistic scheduling: The notion of opportunistic scheduling was introduced by Knopp and Humblet in [4] by proposing the *MaxRate* scheduler. MaxRate exploits multiuser diversities in wireless channel by scheduling the user with the highest transmission rate in each frame. The greedy behavior of MaxRate leads to unfairness among users with heterogeneous channels. Therefore, MaxRate is rarely used or proposed as a solution, whereas it is commonly referred to as an upper bound for the achievable throughput. Furthermore, MaxRate does not define a strategy to break scheduling ties, and common implementations use random tie-breaking [10].

Some of the proposed opportunistic schedulers can provide certain notion of fairness at the expense of reducing capacity. For instance, *Proportional Fair* (PF) represents the state-of-the-art opportunistic scheduling with fairness constraints. PF gives priority to users with relative good channel quality and who have received less throughput in the past, i.e., PF uses the r.v. $R_n(t)/\mu_n(t)$ as scheduling priority, where $R_n(t)$ is the rate achievable in the current frame, and $\mu_n(t)$ is the average throughput received by user n and computed via a low pass filter. The PF scheduler, patented by Qualcomm for High Data Rate (HDR) system 1xEV-DO [5], was shown to maximize the aggregate logarithmic throughput, not the cumulative throughput [8]. While the network capacity is fully achievable in case of homogeneous users [16], this is not the case in presence of multi-class flows [9]. Other opportunistic schedulers, e.g., *MaxWeight* [17] and *Exponential rule* [18], take scheduling decisions based on a metric that combines queue sizes and transmission rates. These schedulers do not achieve the maximum possible throughput.

c) Tie-breaking: Some researchers however realized that advanced tie-breaking can be used to improve performance. In [10], Neely proposed to break the ties in favor of the user with the longest queue. The authors of [19] proposed a modification of the MaxWeight scheduler, which at every frame serves the user with highest product of head-of-line packet's waiting time and actual transmission rate, with ties broken in favor of older packets. In [20], the authors propose to use oldest-first tie-breaking rule in which the scheduler breaks ties based on the life-time of the flows (i.e., packet waiting time in the queue). For flow-level models where users randomly arrive and depart upon completion, [9] and [21] proposed prioritization of flows with higher departure probability in tie-breaking. Indeed, [22] proved that opportunistic schedulers fail to achieve fluid-optimality unless the tie-breaking prioritizes such users.

In contrast to our work, existing proposals do not exploit D2D communications and tie-breaking in opportunistic scheduling to improve fairness while achieving maximum throughput.

III. System model

In this section we present our D2D-based opportunistic scheduling system that can be leveraged to improve both throughput and fairness in the system.

A. Connections

We consider a cellular network with N persistent connections of users to a base station through dedicated wireless channels. In the following, we focus on downlink communications, though the model is applicable also to synchronized uplink communications. The base station operates in a synchronous time-slotted way, and its task is to schedule connections for transmission in every *frame* $t = 0, 1, \ldots$. We also assume that the base station has a queue for storing packets to be delivered to each connection, and queues are never empty (fully backlogged assumption), so that we can evaluate the behavior of the system in *saturation*, i.e., under the worst scheduling operational conditions.

The connection channels are *heterogeneous*, i.e., *not* satisfying the i.i.d. assumption diffused in previous literature. The channel in between the base station and connection n is characterized by stationary Rayleigh fading. Therefore, the SNR for connection n can be described as a random process $C_n(t)$ with mean SNR γ_n, and the Cumulative Distribution Function (CDF) of the SNR has the following expression:

$$F_n(z) = \left[1 - e^{-\frac{z}{\gamma_n}}\right], \quad z \geq 0. \tag{1}$$

We assume that the information available at the base station corresponds to the steady-state distribution of SNR. Note that, for systems in which the possible channel conditions are discrete, existing patents [23]–[25] propose to keep track of historical observations of the SNR of every connection in order to provide an estimate of the steady-state distribution of SNR.

The instantaneous achievable rate of connection n at slot t, $R_n(t) = r_k$, depends on the adopted Modulation and Coding Scheme (MCS) $k = 1, 2, \ldots, K$. We assume that the actual MCS for connection n at slot t is selected as a function of the instantaneous SNR $C_n(t)$, i.e.:

$$R(t) = r_k \iff MCS_n(t) = k \iff C_n(t) \in [c_k; c_{k+1}[\,, \tag{2}$$
$$0 = c_1 < c_2 < \cdots < c_K < c_{K+1} = \infty,$$
$$0 = r_1 < r_2 < \cdots < r_K.$$

Therefore, the probability $p_{n,k}$ that a scheduled connection n receives data encoded according to the k-th MCS is:

$$p_{n,k} = \int_{c_k}^{c_{k+1}} dF_n(z) = e^{-\frac{c_k}{\gamma_n}} - e^{-\frac{c_{k+1}}{\gamma_n}}. \tag{3}$$

We assume that the MCS selection is perfect (transmissions are affected by negligible error rate), so that we ignore retransmission mechanisms. Eventually, we consider a system with no power control, which is typical for realistic downlink transmission schemes.

Table I shows a list of possible modulation and coding schemes for LTE-like networks [26], their coding rate, and the SNR threshold (in *dB*) that has to be reached to achieve a negligible error rate. The Table also contains the net transmission rate, in bits per symbol, achieved with each MCS. The Implementation Margin (IM) in Table I is a value that represents the noise due to non-ideal receiver. In our simulation, MCS thresholds c_k include both SNR and IM.

For the sake of tractability, in what follows we assume that mobile users belong to one of three predefined SNR *classes*, which correspond to *poor*, *average*, and *good* average SNR. The designated average SNR for different classes are chosen in a manner that the mean achievable rates for *poor*, *average*, and *good* users are 20%, 50%, and 80% of the maximum transmission rate achievable in the system, respectively. Therefore, with the MCS values reported in Table I and the assumed Rayleigh fading model, the SNR to be used in Eq. (3) for *poor, average* and *good* users is $\gamma_n = 7\,dB$, $16\,dB$, $23\,dB$, respectively.

TABLE I
MODULATION AND CODING SCHEMES AND THEIR THRESHOLDS

Modulation	**Coding Rate**	**SNR** (dB)	**IM** (dB)	**SNR+IM** (dB)	**Bits per symbol**
QPSK	1/8	-5.1	2.5	-2.6	0.25
	1/5	-2.9		-0.4	0.4
	1/4	-1.7		0.8	0.5
	1/3	-1		1.5	0.67
	1/2	2		4.5	1
	2/3	4.3		6.8	1.3
	3/4	5.5		8.0	1.5
	4/5	6.2		8.7	1.6
16QAM	1/2	7.9	3	10.9	2
	2/3	11.3		14.3	2.66
	3/4	12.2		15.2	3
	4/5	12.8		15.8	3.2
64QAM	2/3	15.3	4	19.3	4
	3/4	17.5		21.5	4.5
	4/5	18.6		22.6	4.8

B. Scheduling of Clusters of Users

In the previous discussion we had implicitly associated every connection with a single user. However, mobile users may form clusters using cooperative D2D communications, in which only one of the users, namely the *cluster head*, connects at a given frame to the base station and relays traffic for the other users. A cluster is formally defined as follows:

Definition 1. (Cluster) A cluster is a group of mobile users that can communicate with each other using an acceptable data rate, typically more advantageously (in some sense that may depend on each user) than with the cellular base station. Only one cluster member, namely the cluster head, is allowed to receive data from the base station within each cellular transmission frame. The downlink traffic received at the cluster head can belong to any of the cluster members.

The scheduling algorithm is MaxRate, i.e., the cluster that contains the user with the highest MCS is scheduled, so we propose to operate clusters in opportunistic way: (i) the cluster head can change on a per-frame basis, as it is opportunistically selected as the cluster member with the highest current MCS rate; (ii) an entire cluster is scheduled as an individual user whose MCS is the highest among members; (iii) the cluster head relays the downlink packets to the final destination (intra-cluster communications) on a secondary wireless interface, using D2D communications. Therefore, in this work, a *connection* n is a cluster (also indicated as CL_n) composed by m_n mobiles, and its instantaneous SNR is the *highest* SNR among the mobile users composing the cluster. In particular, the probability that a scheduled connection n (i.e., cluster CL_n) receives data encoded according to the k-th MCS can be written similarly to Eq.(3) :

$$p_{\mathrm{CL}_n,k} = \int_{c_k}^{c_{k+1}} dF_{\mathrm{CL}_n}(z), \tag{4}$$

where $F_{\mathrm{CL}_n}(z)$ is the CDF of the maximum of m_n random variables representing the SNR values of each of the m_n mobiles forming cluster CL_n:

$$F_{\mathrm{CL}_n}(z) = \prod_{j \in \mathrm{CL}_n} F_j(z) = \prod_{j \in \mathrm{CL}_n} \left(1 - e^{-\frac{z}{\gamma_j}}\right), \; z \geq 0. \tag{5}$$

For simplicity of notation, we omit the "CL" index in the formulas in the reminder of this paper, so that expressions like $p_{n,k}$ and $F_n(z)$ can be equivalently used for connection n and cluster CL_n. Similarly, we will use the notation "connection n" to address either the mobile user n or the cluster CL_n, according to the context. This use of notation stems from the fact that, in our system, a cluster is scheduled as if it were a user, and the only thing formally differentiating users from clusters is the structure of the SNR CDF (Eq. (1) holds for a single user, while Eq. (5) holds for an entire cluster).

As detailed in Subsections III-C, IV-B and V-B, scheduling clusters instead of users not only brings advantages in terms of system throughput, but also in terms of fairness. However, we will mainly focus on inter-cluster fairness, since the actual per-user fairness depends on the way resources are shared within a cluster. Accordingly, when we refer to per-user throughput and fairness, we assume that clusters resources are divided equally between cluster members. Note that user throughput unfairness due to heterogeneous channel qualities within the same cluster is smoothed by the adopted cooperative D2D communications mechanisms. However, the particular mechanism to manage intra-cluster fairness is left out of the scope of this manuscript. Here, we rather focus on studying possible gains in throughput and in fairness among clusters.

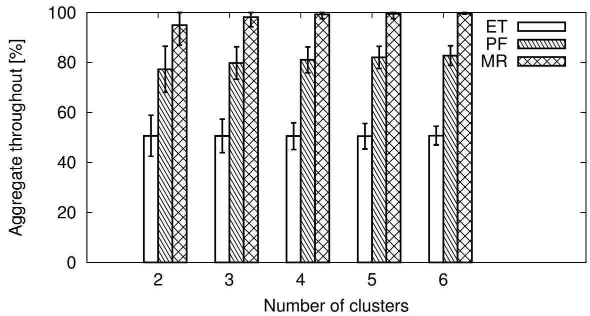
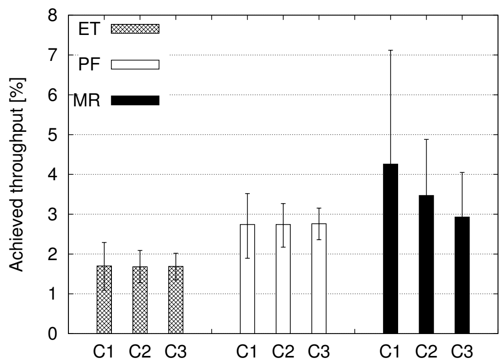
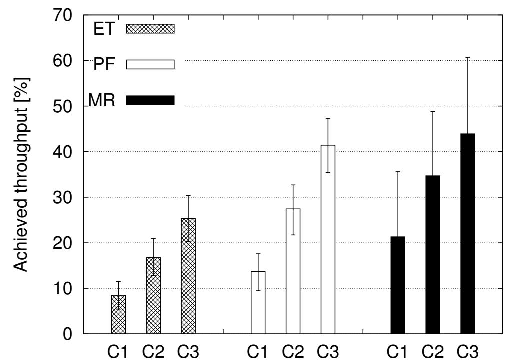


Fig. 2. Throughput comparison (average and standard deviation) of clusters of 1 to 10 users, with uniform quality distribution.



(a) Throughput per user.

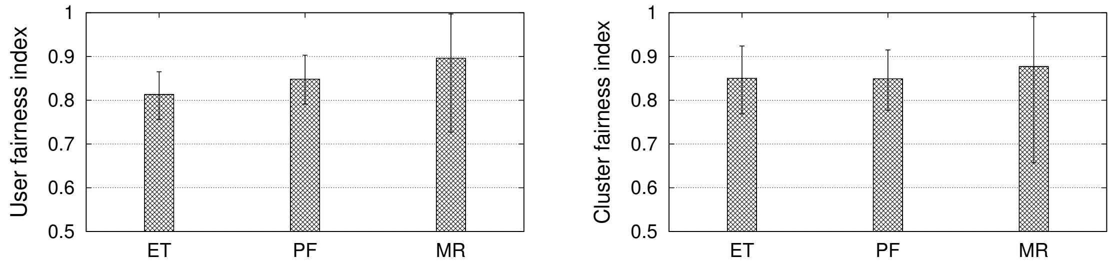


(b) Throughput per cluster.

Fig. 3. Throughput under different schedulers (average plus $5th$ and $95th$ percentiles), assuming resources are divided equally among cluster members, and ties are broken at random.

C. Impact of Clustering on System Performance

The potential clustering gain versus the conventional cellular architecture is discussed here. First, we illustrate the incentive for clustering with simple numerical calculations that show the average clustering gain. We assume that MaxRate schedules clusters and breaks ties at random (i.e., any cluster/connection has the same probability to be selected when a tie occurs). We refer to this particular version of MaxRate as MR. Second, we evaluate throughput and fairness performance. We evaluate the impact of clustering against two baseline schedulers. The first scheduler is *Equal Time* (ET), a simple and largely deployed round robin scheduler which guarantees the same fraction of airtime to each user. The second scheduler that we consider here is PF. In the figures presented in this subsection, results are averaged over 2000 random instances, and user qualities are uniformly distributed among *poor*, *average*, and *good*.



(a) User fairness index under different schedulers. (b) Cluster fairness index under different schedulers.

Fig. 4. Fairness achieved under different schedulers (average plus $5th$ and $95th$ percentiles), with randomized tie-breaking.

For reference, Fig. 2 shows the difference in throughput achieved by ET, PF[1] and MR schedulers as a function of the number of clusters in the network. Cluster sizes are chosen at random, ranging from 1 to 10 members. Of course, the throughput of ET and PF only depends on the number of mobile users and their channel qualities, but we keep using the number of clusters as reference. Interestingly, MR can double the throughput of ET and outperform PF by more than 20%. Most importantly, MR can nearly achieve 100% of the achievable throughput.

We now zoom into the performance experienced in the different clusters. Specifically, we consider a fixed topology, which is the simple one depicted in Fig. 1. Clusters C1, C2 and C3 share the same base station and have 5, 10 and 15 mobile users, respectively. Fig. 3(a) shows the average throughput achieved by mobile users belonging to different clusters, when the MR is adopted to schedule the three clusters. In the Figure, throughput achieved by a cluster is distributed equally by the mobile users forming the cluster, so it represents the average per-user throughput. The aggregate throughput per cluster is shown in Fig. 3(b). For reference, Fig. 3 also reports the throughput achieved with ET and PF schedulers without clustering (per-cluster throughput is then computed as the sum of throughputs achieved by each member separately). The high variability exhibited by MR is a drawback due to its greedy behavior (i.e., its random tie-breaking strategy), and to the occurrence of unbalanced clusters in our simulations (e.g., clusters with only *good* users will achieve extremely high throughput as compared to clusters

[1]PF results are obtained by simulating a scheduling process in which the average user throughput is computed with an autoregressive filter with exponential time constant equal to 1000 frames. However results computed with time constant in the range 50 to 5000 do not significantly differ.

with only *poor* users). Furthermore, clustering helps in terms of *user* fairness, as shown in Fig. 4(a), where the Jain's fairness index [27] among users is graphically depicted. As it can be seen in the Figure, ET and PF are both outperformed by MR, in which throughput is sensibly higher and unfairness is halved. It is interesting to observe that our clustering proposal not only increases the throughput and fairness between users, but it also increases, on average, the fairness index among *clusters*, see Fig. 4(b). However, the variability shown by a pure MaxRate approach with random tie-breaking (MR) is high, which opens to great potentials for improvements.

IV. Maximal Fairness with MaxRate Scheduling with Two Connections

Maximum throughput is achieved in our setting by using MaxRate, which in each frame transmits data to a connection with the highest instantaneous SNR, i.e., the process of selected connection $A(t)$ must satisfy for every t that $A(t) = n$ implies $R_n(t) \geq R_m(t)$ for all connections $m = 1, \ldots, N$. Therefore, by definition, MaxRate is throughput-optimal, and so is our proposal.

The objective of this Section is to study when it is possible, and how, to achieve the *perfect* fairness given that the scheduler be MaxRate. We focus on fairness in the sense of equalizing the expected time-average throughput across connections, independently of their average channel quality. The only degree of freedom that the system offers to play with fairness consists in the occurrence of *ties* in the scheduling mechanism, which is frequent in systems using only few discrete MCS values. This degree of freedom can be exploited by designing a *tie-breaking rule* to employ if at least two connections compete for scheduled with the same highest instantaneous transmission rate. Formally, we use the following definitions:

Definition 2. (Best set $\mathcal{M}$ and best MSC κ) $\mathcal{M}(t, \kappa(t))$ is the set of connections that can be scheduled with the κ-th MCS in frame t, and κ is the best MCS that can be used in the system in frame t, according to the SNR of the connections. We will use $\mathcal{M}$ as short for $\mathcal{M}(t, \kappa(t))$.

Definition 3. (Tie) A tie occurs when, in a given frame t, two or more connections can be scheduled by the MaxRate mechanism with the same MCS$= \kappa$, which is the best possible MCS in the system at that scheduling epoch, that is: $|\mathcal{M}(t, \kappa(t))| > 1$.

Definition 4. (Tie-breaking) A tie-breaking mechanism is a procedure to select exactly one connection $i \in \mathcal{M}(t, \kappa(t))$ to be scheduled when a tie occurs at time t.

In what follows, we first study the two-connection case, for which we give complete answers and which serves to develop fundamental intuition. Subsequently, we show how clustering can be beneficial in achieving perfect fairness via tie-breaking, without paying in terms of throughput. In Section V, inspired by the results achieved for two connections, we extend our approach to the multi-connection case.

A. Analysis of the Two-Connections Case

In many of the arguments we will rely on the fact that the expected long-term fairness (throughput distribution over an indefinitely long interval) is equivalent to the expected one-slot fairness (average per-slot throughput distribution), due to the stationarity channel assumption we made earlier. Let us denote by $Q_{n,k} := \sum_{l=1}^{k-1} p_{n,l}$ the probability that connection $n \in \{1, 2\}$ has an MCS strictly worse than k. Note that $Q_{n,1} = 0$. We further define the following quantities:

$$R^{(1)} := \sum_{k=2}^{K} r_k p_{1,k} Q_{2,k}, \tag{6}$$

$$R^{(2)} := \sum_{k=2}^{K} r_k p_{2,k} Q_{1,k}, \tag{7}$$

$$R^{(X)} := \sum_{k=1}^{K} r_k p_{1,k} p_{2,k}, \tag{8}$$

which represent the expected (both one-slot and time-average) transmission rates in the following three cases: Eq. (6) expresses the rate of connection 1 when it has an MCS strictly better than connection 2; Eq. (7) is for connection 2 having an MCS strictly better than connection 1; and Eq. (8) is for the case of tie. Note that the aggregate throughput of the system under MaxRate scheduler is equal to $R^{(1)} + R^{(2)} + R^{(X)}$.

In the following proposition we give a sufficient and necessary condition for a scheduler that achieves both maximal throughput and fairness.

Proposition 1. *The MaxRate scheduler can achieve both one-slot and time-average fairness if and only if*

$$\left|R^{(1)} - R^{(2)}\right| \leq R^{(X)}. \tag{9}$$

The proof is given in Appendix A. It is worth to discuss when such a condition might hold. Indeed, there are some intuitive sufficient conditions stated next, which are independent of the transmission rates r_k.

Proposition 2. *The MaxRate scheduler can achieve both one-slot and time-average fairness if any of the following conditions hold:*

1) $p_{1,k} = p_{2,k}$ *for all* $k \geq 2$ *(i.e., the channels of the two connections are statistically equal);*
2) $|p_{1,k}Q_{2,k} - p_{2,k}Q_{1,k}| \leq p_{1,k}p_{2,k}$ *for all* $k \geq 2$*;*
3) $p_{1,k} \geq p_{2,k}$ *for all* $k \geq 2$ *and* $p_{2,K} \geq 1/2$*;*

The proof is presented in Appendix B. Moreover, there may be weaker conditions which make it likely that fairness be achievable. For instance, if one of the following conditions hold, perfect fairness is achievable:

1) $p_{1,k}p_{2,k}$ is large enough for all k large enough;
2) $|p_{1,k}Q_{2,k} - p_{2,k}Q_{1,k}|$ small enough for all k large enough;
3) probabilities $p_{n,k}$ for all k large enough are approximately equal for the two connections;
4) the expression $p_{1,k}Q_{2,k} - p_{2,k}Q_{1,k}$ often changes sign as k grows.

Finally, taking into account that transmission rates r_k grow somewhat exponentially with k (see Table I), it is much more important that the two connections be statistically similar in the upper MCS range rather than in the lower MCS range.

Let us define now the *MaxRate-MaxFair scheduler* for two connections, by introducing a bias in the tie-breaking rule of the MaxRate scheduler as follows:

Definition 5. (MaxRate-MaxFair scheduler) In case a tie occurs under MaxRate scheduling, serve connection 1 with probability $\alpha^{(X)}$ and serve connection 2 with probability $1-\alpha^{(X)}$, where

$$\alpha^{(X)} := \frac{1}{2} + \frac{R^{(2)} - R^{(1)}}{2R^{(X)}}. \tag{10}$$

Moreover, if $\alpha^{(X)} \notin [0,1]$, then it is not a proper probability value, thus we cut such values off:

$$\begin{cases} \text{if } \alpha^{(X)} < 0 \text{ then } \alpha^{(X)} := 0; \\ \text{if } \alpha^{(X)} > 1 \text{ then } \alpha^{(X)} := 1. \end{cases} \tag{11}$$

The following proposition establishes when $\alpha^{(X)}$ is a proper probability value, so that no cut-off is needed. The proof is immediate, therefore we omit it.

Proposition 3. *Condition* (9) *is equivalent to* $\alpha^{(X)} \in [0,1]$ *as defined in* (10).

Using MaxRate-MaxFair, connection 1 receives $R^{(1)} + \alpha^{(X)} R^{(X)}$, while the throughput of connection 2 is $R^{(2)} + (1-\alpha^{(X)}) R^{(X)}$. Such a throughput distribution is the fairest possible, and the aggregate is maximum, as stated in the following Proposition, which is the main result of this Section and validates the MaxRate-MaxFair name of the above scheduler.

Proposition 4. *If* (9) *holds, then the MaxRate-MaxFair scheduler achieves maximum throughput, and both one-slot and time-average fairness. If* (9) *does not hold, then the MaxRate-MaxFair scheduler achieves maximum throughput, and the difference between individual throughputs is the minimum achievable with tie-breaking schemes.*

The proof is presented in Appendix C. According to Proposition 4, when condition (9) does not hold, the scheduler still achieves maximum throughput, but will not be perfectly fair anymore. Nevertheless, the difference between individual throughputs will be the minimum possible, and may significantly outperform randomized tie-breaking. Note that, from the proof of Proposition 4, it follows that, using the Jain's fairness index as metric [27], the MaxRate-MaxFair scheduler achieves the smallest possible distance from the perfectly fair throughput distribution. In fact, the Jain's fairness index is maximized when differences are minimized. The result is formalized in the following corollary, which justifies the name we selected for the scheduler.

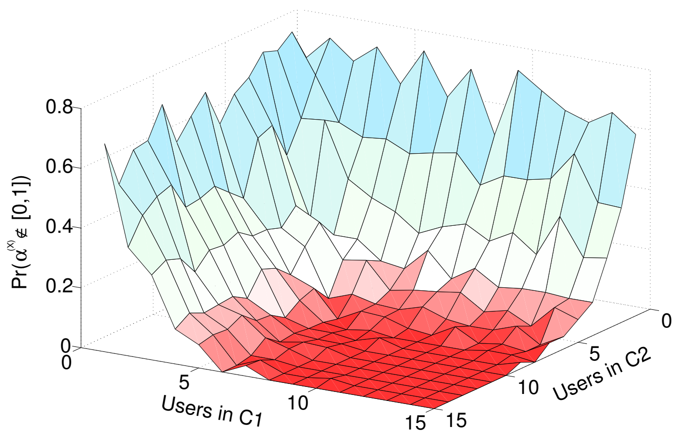


Fig. 5. The probability that perfect fairness cannot be achieved, i.e., $\alpha^{(X)} \notin [0, 1]$, vanishes as the cluster sizes grow.

Corollary 1. *MaxRate-MaxFair scheduler achieves the highest possible Jain's fairness index achievable by means of any tie-breaking mechanism.*

B. Impact of Cluster Composition on the Two-Connection Case

Under MaxRate scheduling, both user-based or cluster-based scheduling is throughput-optimal. However, the advantage of clustering consists in the possibility to re-distribute the cluster throughput not only among clusters, but also among cluster members, thus yielding potentially higher fairness levels. We exemplify this effect by considering the specific case of *MaxRate-MaxFair* with two clusters of possibly different sizes. Each cluster can be regarded as a single connection, and the exact analysis of Subsection IV-A applies. Specifically, thanks to clustering with opportunistic cluster head selection, SNR statistics of scheduled connections of both clusters are improved, which decreases the probability of $\alpha^{(X)}$ falling out of range $[0, 1]$.

Fig. 5, which is the result of $20,000$ random instances for two clusters of random size and composition, is in line with our intuition on the effect of cluster size on $\alpha^{(X)}$. The Figure shows the probability that $\alpha^{(X)}$ be outside the acceptable range $[0, 1]$, when the perfect fairness can not be achieved. The Figure reveals that perfect fairness can be achieved almost surely when both clusters consist of more than 5 users. The probability of fairness non-achievability radically increases as the cluster size drops below 5 users, since the average cluster qualities can be very unbalanced and yield large $|R^{(1)} - R^{(2)}|$. In contrast, when clusters are large enough (i.e., with more $5-10$ members), the fact that each cluster head exhibits the highest SNR in its cluster makes the probability to use the best MCS practically 1, and thus $|R^{(1)}-R^{(2)}|$ approximates 0, while increasing the probability of ties. Therefore, condition (9) is met with high probability.

V. Maximal Fairness with MaxRate Scheduling with Multiple Connections

Having developed fundamental intuition based on exact results for the case of two connections, we now set out to design MaxRate schedulers for generic number of connections N in a set $\mathcal{N}$, achieving better fairness than with randomized tie-breaking. Extending the analytical approach from the two-connection case would require $(N-2)2^{N-1}+1$ tie-breaking parameters for all possible ties of $2, 3, \ldots, N$ connections, which grows too fast to be implementable $(1, 5, 17, 49, \ldots)$. Instead, we focus on scalable solutions that rely on at most N tie-breaking parameters.

A. WRR Tie-breaking

We design a scheduler in which tie-breaking is resolved as if Weighted Round Robin (WRR) was implemented. Thus, we assume that for each connection n there is a non-negative parameter α_n used as follows.

Definition 6. (WRR Tie-breaking) If, at a given scheduling epoch t, $\mathcal{M}$ is the set of connections that are currently tied in the highest MCS (see Definition 2), then the probability (or, the average fraction of time) that connection $n \in \mathcal{M}$ is served in such situations is as follows:

$$\frac{\alpha_n}{\sum_{m\in\mathcal{M}} \alpha_m}, \quad \alpha_m \geq 0 \quad \forall m \in \mathcal{M}. \tag{12}$$

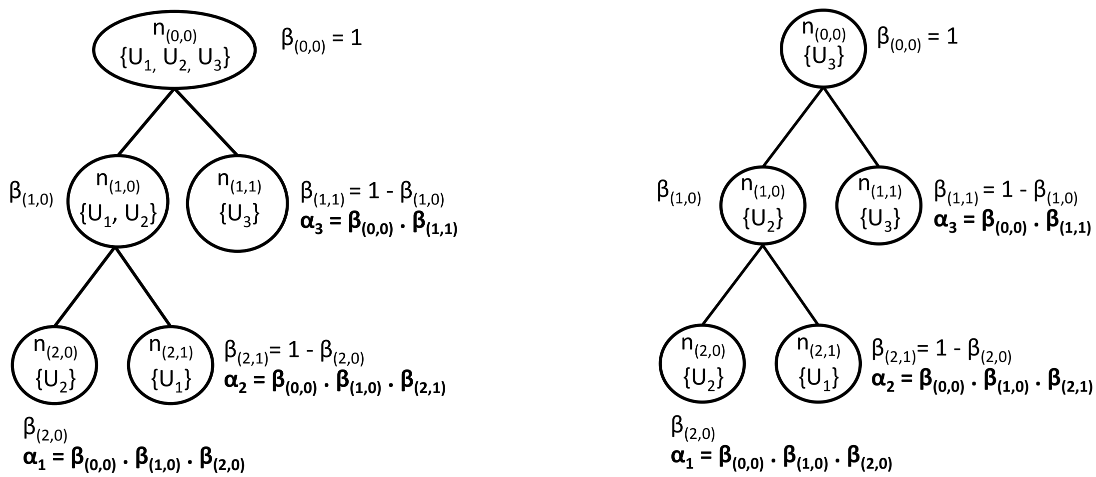
The expected throughput of connection n under MaxRate with WRR tie-breaking is then:

$$\sum_{k=1}^{K} r_k \sum_{\mathcal{M}\ni n} \left[\frac{\alpha_n}{\sum_{m\in\mathcal{M}} \alpha_m} \prod_{m\in\mathcal{M}} p_{m,k} \prod_{m\notin\mathcal{M}} Q_{m,k} \right], \tag{13}$$

where $\mathcal{M} \ni n$ denotes any set of connections that includes n.

Using (13) it is, however, intractable to obtain values of α_n which would equalize the expected individual throughput of all connections, since it leads to a system of non-linear equations. Hence, in the following we propose heuristics to obtain α_n.

1) Heuristic 1: Best Leaf First (BeLF): This method is based on the results of the two-connection case, and on the use of binary trees. We have shown in Section IV-A the exact way to compute α for two connections in a tie. Now, two connections can represent two mobile users, as well as two clusters. More in general, the approach of Section IV-A is valid for any two *groups* of users for which an SNR CDF is available to compute the probabilities to use the various MCS values. Therefore, we can use the results presented in Section IV-A to compute the optimal tie-breaking probability for any two disjoint and not empty user groups covering the entire set $\mathcal{N}$, say subsets $\mathcal{N}_{1,0} \neq \emptyset$ and $\mathcal{N}_{1,1} \neq \emptyset$, $\mathcal{N}_{1,0} \cup N_{1,1} = \mathcal{N}$, $\mathcal{N}_{1,0} \cap N_{1,1} = \emptyset$. Let us call $\beta_{1,0}$ and $\beta_{1,1} = 1 - \beta_{1,0}$ the tie-breaking probabilities of $\mathcal{N}_{1,0}$ and $\mathcal{N}_{1,1}$, respectively. These priorities are computed as per Eqs. (10) and (11) given in Definition 5. Any of the subsets grouping at least two users can be further split into two subsets, e.g., if $|\mathcal{N}_{1,0}| \geq 2$, $\exists \mathcal{N}_{2,0} \neq \emptyset, \mathcal{N}_{2,1} \neq \emptyset$, for which $\mathcal{N}_{2,0} \cup N_{2,1} = \mathcal{N}_{1,0}$, $\mathcal{N}_{2,0} \cap N_{2,1} = \emptyset$. To these smaller subsets, we can associate again two tie-breaking probabilities $\beta_{2,0}$ and $\beta_{2,1} = 1 - \beta_{2,0}$, computed as per Definition 5. Each subset with at least two users can be recursively split into two subsets, and each subset receives a tie-breaking probability $\beta_{i,j}$, where i is the level of recursion, and j is a sequential index within a recursion level. This binary splitting procedure can be represented with a binary tree, as shown in Fig. 6(a). The tree root $n_{0,0}$ represents the entire network $\mathcal{N}$, and its tie-breaking probability is formally set to 1. Leaves represent connections (either individual mobile users or clusters), and each node $n_{i,j}$ at level i in the tree represents the group of connections that appear as leaves in the branches originating at that node. We finally associate a WRR priority α_n to each of the N leaves of the tree: since each $\beta_{i,j}$ represents a conditional tie-breaking probability (given that there is a tie between two groups), the WRR priority of a connection is computed from the corresponding leaf as the product of the $\beta_{i,j}$ values on the path from the root to the leaf. With the above procedure, the sum of WRR priorities α_n is exactly 1, so that they can be directly interpreted as tie-breaking probabilities for a multi-connection case.



(a) Computation of α_n with BeLF. (b) Computation of α_n with WoLF.

Fig. 6. Example of BeLF and WoLF with 3 users (numbers in brackets represent users, U_1 being the best user and U_3 being the worst).

The proposed heuristic might not work well when two leaves in the same branch are associated to users with very different average channel qualities, e.g., in very heterogeneous network conditions. This effect is due to the adoption of an expression similar to Eq. (5) for the computation of the SNR CDF associated to a node in the tree, i.e., the SNR of a node in the tree corresponds to the highest SNR among the users (leaves) connected to that node. In particular, at a given node of the binary tree, the presence of a branch without users with statistically good channel, namely *good* users, is kept "hidden" by the presence of another branch departing from the same node in which *good* users are present. Thus, qualitatively speaking, the grouping mechanism described here fails in distinguishing groups that differ in the number of poorly performing members.

2) Heuristic 2: Worst Leaf First (WoLF): The previous discussion suggests an alternative way of building priorities by means of a binary tree. In particular, one could proceed as for the BeLF heuristic, but consider each node in the tree as a group represented by the *worst* channel quality among all users associated to leaves on the branches originating at that node (see Fig. 6(b)). Therefore, the presence of a user with statistically poor channel on a leaf provokes a shift in the distribution of priorities towards the branch that contains that user.

3) Heuristic 3: Fair Individual Share (FISh): Another possibility is to compute α_n for each connection n in the system as the $\alpha^{(X)}$ of a two-connection system (see Eq. (10)) in which connection n competes with the rest of connections. In this case, we assume that the target of connection n is to achieve a portion $1/N$ of the cell throughput.

Following the ideas from the two-connection case, we first define $Q_{-n,k}$ as the CDF for the best MCS of all the connections except connection n, $Q_{-n,k} := \prod_{m=1,m\neq n}^{N} Q_{m,k}$, and $p_{-n,k}$ as the probability that at least one of the connections (except connection n) is in MCS k and no other connection is in a better MCS, formally, $p_{-n,k} = Q_{-n,k+1} - Q_{-n,k}$. Then, (6) to (8) can be rewritten as follows, for $n = 1 \dots N$:

$$R^{(n)} := \sum_{k=2}^{K} r_k p_{n,k} Q_{-n,k}, \tag{14}$$

$$R^{(-n)} := \sum_{k=2}^{K} r_k p_{-n,k} Q_{n,k}, \tag{15}$$

$$R_n^{(X)} := \sum_{k=1}^{K} r_k p_{n,k} p_{-n,k}. \tag{16}$$

The total cellular throughput of MaxRate is $R_{Tot} = R^{(n)} + R^{(-n)} + R_n^{(X)}$, which is the same for all values of n, as it can be easily verified.

Proposition 5. *The MaxRate scheduler for a system with $N \geq 2$ connections can achieve both one-slot and time-average fairness if and only if*

$$\left| R^{(n)} - R^{(-n)} \right| \leq R^{(X)}, \quad \forall n = 1 \dots N. \tag{17}$$

The proof of the Proposition 5 derives from the proof of Proposition 1.

The value of the tie-breaking probability α_n is computed from the following equation:

$$R^{(n)} + \alpha_n R_n^{(X)} = \frac{1}{N} R_{Tot}. \tag{18}$$

The resulting value of α_n is then:

$$\alpha_n = \frac{1}{N} + \frac{\sum_{k=1}^{K} r_k \left[p_{-n,k} Q_{n,k} - (N-1)\, p_{n,k} Q_{-n,k}\right]}{N \sum_{k=1}^{K} p_{n,k} p_{-n,k} r_k}. \tag{19}$$

However, when the expected transmission rate of connection n is strictly higher than the average fair individual share (i.e., $R^{(n)} > \frac{1}{N} R_{Tot}$), then α_n is negative, which is not acceptable for the WRR scheduling mechanism. This corresponds to a situation in which the amount of resources in ties are not enough to equalize the connection throughputs without loss of throughput maximality. Therefore, we propose the following transformation which preserves the order of α_n, i.e., preserves the *priority list* among connections:

$$\forall n \in \mathcal{N}, \quad \alpha_n := \alpha_n - \min_{m \in \mathcal{N}} \alpha_m. \tag{20}$$

Note that, since we use α_n values as described in (12), the proposed transformation is equivalent to normalizing the values of α_n in the interval $[0, 1]$, as if using $\alpha_n := \frac{\alpha_n - \min_{m \in \mathcal{N}} \alpha_m}{\max_{m \in \mathcal{N}} \alpha_m - \min_{m \in \mathcal{N}} \alpha_m}$. We remark that, in practice, we do not need to enforce any transformation if some $\alpha_n > 1$, since WRR normalizes such values.

4) Heuristic 4: Priority Keying (PIKe): Forcing the values α_n in the interval $[0, 1]$ might result in one or more connections not benefitting from tie-breaking at all (i.e., connections with $\alpha_n = 0$). However, considering that α_n represents the excess throughput received by connection n, setting $\alpha_n = 0$ should be allowed only for connections receiving more than the fair share, i.e., connections for which $R^{(n)} > \frac{1}{N} R_{Tot}$. Therefore, we propose a modified version of FISh, namely PIKe, in which priorities α_n are shifted only if negative values are present:

$$\text{if } \min_{m \in \mathcal{N}} \alpha_m < 0, \quad \text{then} \quad \forall n \in \mathcal{N}, \quad \alpha_n := \alpha_n - \min_{m \in \mathcal{N}} \alpha_m. \tag{21}$$

As for the case of FISh, the proposed transformation is equivalent to normalizing the values of α_n in the interval $[0, 1]$. However, the transformation is operated only when there exist negative values of α_n.

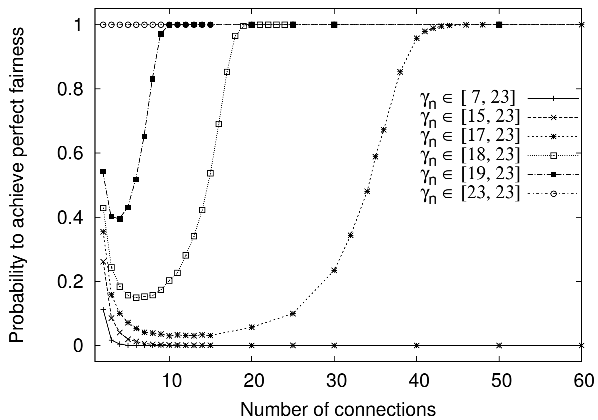


Fig. 7. Probability to achieve perfect fairness with MaxRate without clustering under different levels of connection quality heterogeneity (under different ranges for γ_n).

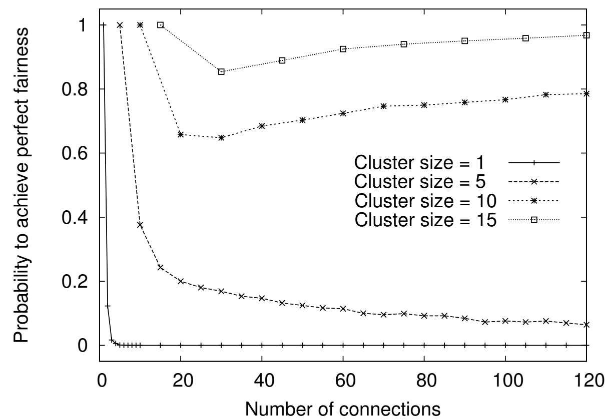


Fig. 8. Probability to achieve perfect fairness with MaxRate with different cluster sizes, under large connection quality heterogeneity ($\gamma_n \in [7, 23]$ *dB*).

In Section VI, we will quantify the level of fairness achieved in the system with the four proposed heuristics. We will also show that, on average, the PIKe heuristic performs better than the others. Although we will use static clustering scenarios to illustrate the advantages of our proposal, we remark that our methodology and findings apply to clusters whose composition varies in time, as some users may turn on/off their devices or migrate to another cluster or another cell. In particular, we note that in presence of *flows* with limited duration, probabilities $p_{n,k}$ described in Session III can be adapted to represent the steady-state probabilities of flow n given the arrival and flow-size distributions.

B. Impact of Clustering on WRR Tie-breaking with Multiple Connections

Perfect fairness could be impossible to achieve under MaxRate scheduling due to the heterogeneity of user's channel qualities (see Propositions 1 to 4). However, as mentioned earlier, clustering *enough* users, i.e., as few as $5 - 10$ mobile users, in practice, causes a very high probability to use the highest MCS value only. Therefore, clustering reduces the heterogeneity of the channel quality as observed by a scheduled connection (i.e., a cluster head).

To appreciate the impact of channel heterogeneity and clustering on a system with multiple connections, we depict in Figs. 7 and 8 the probability to achieve perfect fairness as a function of connection quality distribution and cluster size, for a variable number of connections in the system. Fig. 7 shows simulation results using the MaxRate scheduler when the mean SNR γ_n of user n is picked from a uniform distribution.

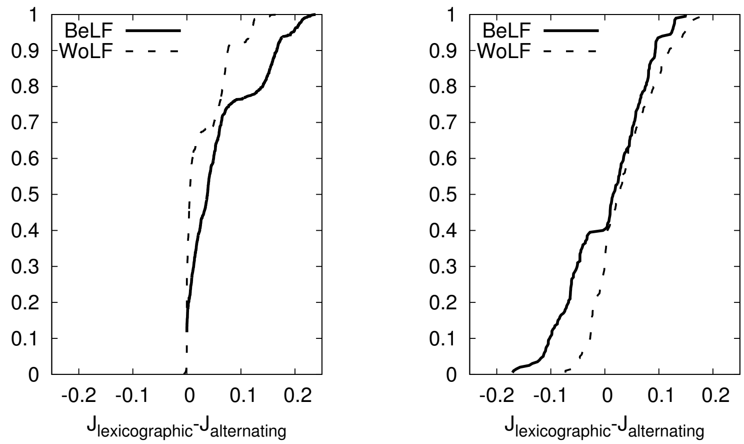
Different intervals for γ_n are considered in the Figure, to show the impact of different degrees of channel heterogeneity. For each interval of γ_n, we tested $10,000$ random instances of $N \in \{1 \dots 60\}$ connections, and checked whether any tie-breaking strategy could lead to perfect fairness or not (*brute force search of the optimal tie-breaking*). Observing Fig. 7, we can deduce that, depending on the interval in which the mean SNR can range, having a few tens of users in the cell can enable perfect fairness via tie-breaking. However, under typical heterogeneous conditions, in which the range for the mean SNR γ_n is several *dB* units, the probability to achieve perfect fairness is almost zero even when the number of users per cell is very high. Therefore, reducing heterogeneity in channel qualities is key to achieve fairness.

The impact of clustering on such quality heterogeneity is shown in Fig. 8 for large levels of heterogeneity ($\gamma_n \in [7, 23]$ *dB*). The Figure illustrates that clustering heterogenous users results in increased probability to achieve perfect fairness under MaxRate scheduling.

Notably, not using clustering makes the probability to achieve fairness practically negligible (see Fig. 8 when Cluster size is 1). In contrast, using small clusters (as few as 5 users) dramatically increases the probability to achieve perfect fairness from $\sim 0\%$ to 10% or more when the number of users in the cell ranges from 20 to 80. Larger cluster sizes (e.g., 10 or 15) further boost perfect fairness achievability to 70% or 90% with a reasonable number of users in the system. Therefore, the potential impact of clustering on the fairness performance is paramount.

VI. Evaluation

In this Section we use numerical evaluation of our D2D-based cluster scheduling proposals. We use the Jain's fairness index to compare the effectiveness of our D2D-based schemes as compared to ET and PF scheduling with unclustered mobile users. As for the throughput, we normalize throughput results in terms of cell capacity. Since our work does not investigate intra-cluster mechanisms, we limit the performance evaluation to the throughput received by the cluster



(a) 3 clusters.

(b) 6 clusters.

Fig. 9. CDF of the difference in the Jains' fairness indexes computed with the lexicographic mapping and with the alternating mapping (Random clusters with size: 1 to 10 users, each of which an be *poor*, *average* or *good* with the same probability).

as a whole. When we refer to per-user throughput, we assume that the total cluster throughput can be equally shared among cluster members, using cooperative D2D communications.

A. *Mapping clusters to leaves in BeLF and WoLF*

Before proceeding with the full evaluation of the heuristics proposed in Section V, let us recall that in our binary tree-based schemes, BeLF and WoLF, the leaves represent the entities to be scheduled. In principle, the order in which leaves in the binary tree are associated to connections (either users or clusters) is not necessarily related to topological considerations. In fact, nodes represent fictitious groups, not necessarily clusters. The only aim of defining fictitious groups consists in allowing a simple computation of WRR tie-breaking priorities for the real entities to be scheduled. However, the way in which connections are grouped in the binary tree affects the achieved fairness level.

Let us refer to connections as clusters, and associate each cluster with a *goodness* metric, which consist in counting the number of *good* users first, then the number of *average* users and eventually the number of *poor* users. Using this metric in our simulations, we have noticed that the highest fairness is achieved under one of two particular mappings. The first is a lexicographic mapping, i.e., connections are sorted from the best to the worst, and mapped in this order onto the leaves of the tree, from left to right. The second mapping consists in sorting connections according to an alternating order, i.e., the best connection first, then the worst, then the second best, followed by the second worst, and so on. Figs. 9(a) and 9(b) depict two examples of CDF of the difference in Jain's indexes achieved by lexicographic and alternating mappings, respectively $J_{lexicographic}$ and $J_{alternating}$. Fig. 9(a) shows that with 3 connections both BeLF and WoLF yield better results with the lexicographic mapping. In contrast, Fig. 9(b) shows that when the number of connections increases to 6, it is not clear which mapping is better, even though the difference is quite limited with very high probability (the CDF grows quite sharply around 0).

However, since we are interested in the potential performance of D2D-based clustering systems, in the following, in order to compare BeLF and WoLF to the other proposed schemes, we will show results computed with the best mapping of clusters to tree leaves, which we found by testing all possible permutations.

B. Comparison of the Heuristics for MaxRate with WRR Tie-breaking

We now compare our proposed MaxRate variants based on the four heuristics introduced to compute the weights for the WRR tie-breaking. Specifically, we compare BeLF, WoLF, FISh, and PIKe in terms of per-cluster fairness. As a reference, we report the fairness indexes achieved by a plain MaxRate scheme where ties between connections are broken randomly (MR in the Figures). We also compare results achieved by ET and PF. Note that, according to the common understanding, PF should have much higher fairness than MaxRate-based approaches [28], while we show that the opposite is true under cooperative D2D communications approaches, in which connections represent clusters. For ET and PF, we numerically simulate the scheduling of single users, then we sum up the throughput of users according to which cluster they belong to.

Fig. 10 reports fairness indexes for the different scheduling schemes for systems with 2 to 6 clusters, each formed by 1 to 10 members. Fairness indexes are shown in terms of box and whiskers plots, reporting minimum and maximum values recorded over the set of simulations performed, the $25th$ and $75th$ percentiles of their values, and the average (the solid dashes in the boxes reported in the Figure). Interestingly, the level of fairness achieved by our proposed schemes is very high, and PIKe achieves as much fairness as PF. Recalling that the throughput

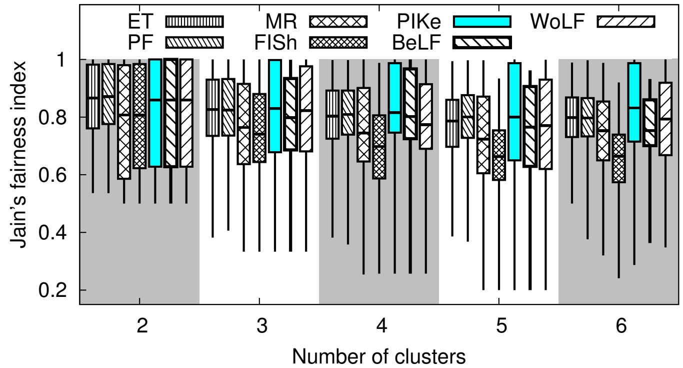


Fig. 10. Comparison of fairness achieved with ET, PF, MR, and MaxRate with WRR tie-breaking (Cluster size: 1 to 10 users).

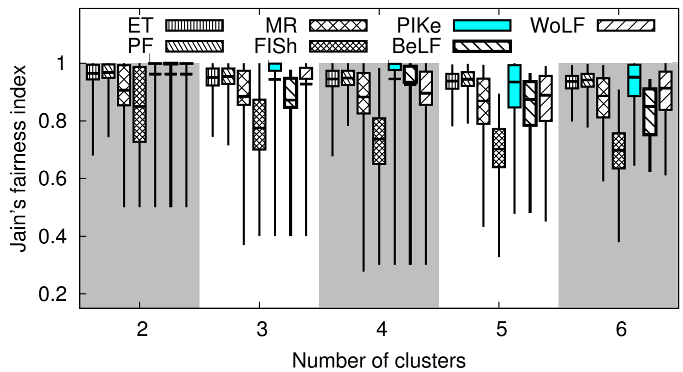


Fig. 11. Comparison of fairness achieved with ET, PF, MR, and MaxRate with WRR tie-breaking (Cluster size: 5 to 10 users).

achieved by PF is much lower than the one achieved by PIKe, this result is very encouraging.

Even more interestingly, simulations accounting for clusters of at least 5 users reveal that PIKe can outperform ET and PF in terms of fairness, as depicted in Fig. 11, where the boxes delimited by the $25th$ and $75th$ percentiles are very close to 1 for the PIKe scheme.

Figs. 10 and 11 show that PIKe, BeLF, and WoLF clearly outperform the MR cluster scheduler, which justifies the work carried out in this manuscript. Comparing MR and PIKe, it can be seen that PIKe reduces the distance from perfect fairness (i.e., 1) by 50%. Note also that FISh achieves significantly poorer performance than the benchmarking policies, especially as the number of clusters increases.

To conclude, we remark that our proposed schemes, and in particular PIKe, are beneficial for both throughput and fairness, which means that they would allow *better worst-case performance* in comparison to ET and PF. Indeed, Fig. 12 shows that the minimum throughput received by a cluster member in the system, using PIKe, is much higher that the one achieved with ET (by

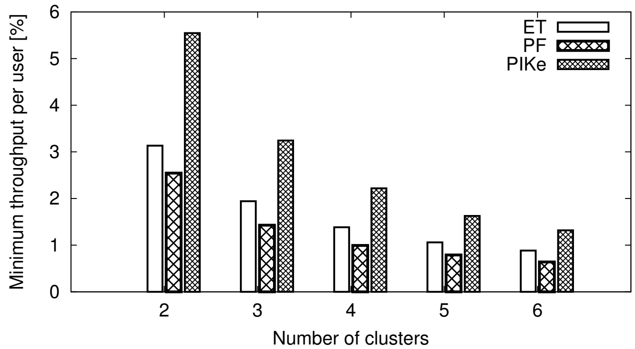


Fig. 12. Minimum throughput attained by a cluster member (average over 2000 simulations—Cluster size: 5 to 10 users).

a factor ~ 1.5 or more), and PF (by a factor ~ 2).

VII. Conclusions

In this manuscript, we have shown how to attain maximal throughput in cellular networks without paying any fairness penalty. Although scheduling ties are usually ignored and random tie-breaking is an accepted practice, we have shown that this practice is rather inefficient. Our simulations indicated that there is a great potential for fairness and throughput enhancement in customized tie-breaking. To achieve this result, we have proposed to leverage scheduling tie-breaking strategies, which are enabled by the use of D2D communications. In particular, we have rigorously formulated a tie-breaking mechanism that achieves maximal fairness *and* maximal throughput for the case of two scheduled connections (e.g., users or clusters). Inspired by such a result, we have proposed four heuristics to extend our results to the generic case of N scheduled connections. Our results confirm that our heuristics, and in particular PIKe, achieve almost perfect fairness and maximal throughput. In our future research we will investigate on the impact of cluster dynamics and flow-size, on how to distribute the throughput efficiently within each cluster and on the overhead associated to such a mechanism.

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APPENDIX A
PROOF OF PROPOSITION 1

Proof: Observe first that $R^{(n)}$ is the minimum achievable transmission rate for user $n = \{1,2\}$ under the MaxRate scheduler, since this user must be served if it has an MCS strictly better than the other user. Moreover, $R^{(n)} + R^{(X)}$ is the maximum achievable transmission rate for user $n = \{1,2\}$ under the MaxRate scheduler, since this user must be served if it has an MCS strictly better than the other user and, in addition, it can be served at most in all the ties. Then, fairness cannot be achieved if either $R^{(1)} > R^{(2)} + R^{(X)}$ or $R^{(2)} > R^{(1)} + R^{(X)}$, which is equivalent to $\left|R^{(1)} - R^{(2)}\right| > R^{(X)}$. In contrast, fairness can be achieved if (9) holds, as shown in (4). Therefore, the equivalence holds. ■

APPENDIX B
PROOF OF PROPOSITION 2

Proof: Item 1) is a special case of item 2), which holds because

$$\left|\sum_{k=2}^{K} r_k \left(p_{1,k}Q_{2,k} - p_{2,k}Q_{1,k}\right)\right| \leq \sum_{k=2}^{K} r_k \left|p_{1,k}Q_{2,k} - p_{2,k}Q_{1,k}\right| . \tag{22}$$

Item 3) can be proved as follows: $p_{2,K} \geq 1/2$ is equivalent to $p_{2,K} \geq Q_{2,K}$, therefore by non-negativity of probabilities it is true that

$$-\sum_{k=2}^{K} p_{2,k-1} Q_{1,k} - \sum_{k=2}^{K-1} p_{2,k} Q_{1,k} \leq p_{2,K} - Q_{2,K}. \tag{23}$$

By adding $Q_{2,K} - Q_{1,K} p_{2,K}$ we have

$$Q_{2,K} - \sum_{k=2}^{K} p_{2,k-1} Q_{1,k} - \sum_{k=2}^{K} p_{2,k} Q_{1,k} \leq (1 - Q_{1,K}) p_{2,K}. \tag{24}$$

By expanding $Q_{1,K}$ and $Q_{2,K}$ we further obtain

$$\sum_{k=2}^{K} p_{2,k-1}(1 - Q_{1,k}) - \sum_{k=2}^{K} p_{2,k} Q_{1,k} \leq p_{1,K} p_{2,K}. \tag{25}$$

Realizing that the first sum equals $\sum_{k=2}^{K} p_{1,k} Q_{2,k}$, we have

$$r_K \sum_{k=2}^{K} (p_{1,k} Q_{2,k} - p_{2,k} Q_{1,k}) \leq r_K p_{1,K} p_{2,K}. \tag{26}$$

For each $k \geq 2$, term $p_{1,k} Q_{2,k} - p_{2,k} Q_{1,k} \geq 0$ since $p_{1,k} \geq p_{2,k}$, therefore

$$\left| \sum_{k=2}^{K} r_k (p_{1,k} Q_{2,k} - p_{2,k} Q_{1,k}) \right| \leq \sum_{k=1}^{K} r_k p_{1,k} p_{2,k}. \tag{27}$$

■

APPENDIX C

PROOF OF PROPOSITION 4

Proof: Consider the MaxRate scheduler with randomized tie-breaking with bias $\alpha \in [0, 1]$ for user 1. Then, the expected time-average and one-slot individual throughputs are $R^{(1)} + \alpha R^{(X)}$ and $R^{(2)} + (1 - \alpha) R^{(X)}$, respectively. It is straightforward to verify that, if (9) holds, plugging $\alpha^{(X)}$ for α the throughput of each user is equal to $\left(R^{(1)} + R^{(2)} + R^{(X)}\right)/2$.

If (9) does not hold, then suppose that $R^{(1)} > R^{(2)} + R^{(X)}$ (case $R^{(2)} > R^{(1)} + R^{(X)}$ is analogous). The difference in the individual throughputs is $R^{(1)} + \alpha R^{(X)} - (R^{(2)} + (1 - \alpha) R^{(X)}) = R^{(1)} - R^{(2)} - R^{(X)} + 2\alpha R^{(X)}$, which is minimized if $\alpha = 0$. Indeed $\alpha^{(X)} = 0$, because of the cut-off of a negative value given by (10). ■