Modeling and Analysis of Opportunistic Routing in Multi-hop Wireless Networks

Amir Darehshoorzadeh*, M. Isabel Sanchez*†‡ and Azzedine Boukerche*
*PARADISE Research Laboratory - School of Electrical Engineering and Computer Science
University of Ottawa, Canada
†IMDEA Networks Institute, Spain
‡Departamento de Ingenieria Telematica, University Carlos III, Madrid, Spain
Email: adarehsh@uottawa.ca, mariaisabel.sanchez@imdea.org, boukerch@site.uottawa.ca

***Abstract*—Opportunistic Routing (OR) takes advantage of the broadcast nature of the wireless medium to increase reliability in communications. Instead of selecting one node as the next-hop forwarder, OR selects a set of candidates to forward the packet. In this way, if one of them does not receive the packet from the source, another candidate will be able to do it; this avoids the need for a re-transmission from the source. To increase the successful delivery ratio, we can increase the size of the candidate set or the number of re-transmissions, but, we must take into account the impact on the use of the network resources. In this paper, we propose a Markov chain as a general model for OR that can be applied to any kind of network topology and any candidate selection algorithm without any constraint. The only input parameters needed are: *i)* the candidate list of each node; *ii)* the link delivery probability between nodes; and, *iii)* the maximum number of re-transmissions in each node. Taking this data into account, our model allows for the evaluation of the performance of different candidate selection algorithms according to different metrics, such as the expected number of transmissions (ExNT), which is one of the most relevant metrics in OR. Our model also enables an evaluation of the influence of the number of candidates, its relation to the number of re-transmissions and how these two parameters together contribute to the successful delivery of data packets.**

I. Introduction

Traditional routing in wireless networks designates just one node as the next-hop forwarder to a given destination. If the selected next-hop does not receive the packet, the source must re-transmit it. Therefore, in traditional routing protocols, all packets travel through a pre-defined path [1], [2]. A link breakage over the selected path forces the finding and establishing of a new path and the re-transmission of data with the subsequent costs in terms of time and bandwidth.

Opportunistic Routing (OR) [3] has been a hot research topic for a few years now. The basic idea behind OR is that it selects the next-hop forwarder on-the-fly, as the packet progresses through the network by taking advantage of the broadcast nature of the wireless medium. In other words, instead of establishing a single next-hop to forward the packets, OR selects an ordered set of nodes (*candidate set*) as the potential next-hop forwarders. The source of a data transmission includes its candidate set in the header of every data packet and broadcasts it. Each of the candidates which receives the packet can act as the next-hop forwarder. Note that each candidate has a priority level which determines the preference for that candidate over the other ones to act as the next-hop forwarder. The priority assignment for candidates follows certain criteria: distance to the destination, hop count or battery constraints [4]. In this way, only the most suitable candidate among those which have received the packet will forward it while the rest of the candidates will discard it.

OR introduces different challenges to the design and evaluation of OR protocols, for instance, the selection of the candidates for the next transmission and the coordination among them to select which one will actually forward the packet. The main aim of an OR protocol is to reduce the expected number of transmissions (ExNT) from source to destination while increasing the probability of reaching the destination.

Most of the research in OR focuses on candidate selection and coordination algorithms, and relies on simulation as the validation tool [5]–[9]. Nevertheless, there is much less research focusing on analytical models in OR to provide the researchers with different aspects and information about the performance that can be achieved using OR. We can establish an analogy between a Discrete Time Markov Chain (DTMC) [10] and OR. Assuming perfect coordination between the candidates, only one candidate will forward the packet at each transmission, and the set of candidates chosen for the transmission is independent of the ones used in previous transmissions. Furthermore, each candidate is used as the next-hop forwarder according to a given probability, so there can be several paths to arrive to the destination. As we can see, the properties of an OR transmission are very similar to a DTMC.

Motivated by the above, in this paper we propose a Discrete Time Markov Chain (DTMC), which allows to analyze the performance of OR in terms of the expected number of transmissions from source to the destination (ExNT), the probability of reaching the destination from the source node, and the distribution of the probability of the number of transmissions. For each node, the candidate set, the link probability to each of them and the maximum number of re-transmissions are inputs to our model. Hence, our model does not require any specific assumptions about the network topology nor the mechanism for selection and prioritization of candidates. To the best of our knowledge, we are the first researchers who investigated the effect of the number of re-transmissions in each node in the case that the packet does not reach any of the candidates. Our model allows to evaluate the impact of a limited number of re-transmissions, and assess whether the use of OR and several candidates can compensate the need for retries. As a result, the use of our model can provide the guidelines to other researchers to evaluate the performance of different candidate selection algorithms, and to investigate the influence of the number of candidates, how it relates to the number of re-transmissions and how these two parameters together contribute to the successful delivery of data packets.

The remainder of this paper is organized as follows: in Section II we present the related work available in the literature which also covers existing analytical models in OR. Section III introduces the Markov model proposed to analyze the performance of OR. This analytical model is validated with a linear topology presented as a case study in Section IV, which illustrates some graphs and results that support our model. Finally, Section V concludes the paper.

II. Related work

Markov processes have been extensively used to model a huge variety of stochastic processes, from chemistry to economics, not limited to the field of network communications. Although they have been applied to queuing theory and wireless networks, to our knowledge, only [11], [12] has applied these processes to Opportunistic Routing (OR). In [11], it is assumed that the packets could be transmitted an infinite times until it reaches to the destination. the contrary, we limit the number of times that a node can re-transmit a packet. There are several reasons to justify this decision. First of all, if the link conditions are not stable enough, every node may need several re-transmissions to successfully deliver the packet, increasing the contention and the delay. Moreover, the strength of OR resides in the use of several candidates to avoid re-transmitting. In addition, after some time, the packet information may not be meaningful. Secondly, as we show in Section IV, a low number of re-transmissions can considerably improve the network performance; therefore, it is not necessary to allow an infinite number of retries.

Another approach [13] is based on the claim that transmitting packets at different rates may increase the throughput of OR, but the communication range associated with different rates must also be taken into account. [14] optimizes candidate selection by minimizing the distance towards the destination, according to a given routing metric. Specifically, the average number of transmissions is selected as the routing metric. The authors proposed two algorithms for finding the optimum candidate list, considering the trade-off between the probability of reaching the destination and the number of transmissions.

In [15] a *random walk* model in a Markov chain is used to analyze the end-to-end transmission cost of different routing mechanisms, being OR among them. Like our model, [15] also focuses on the expected number of transmission as the metric to evaluate performance, as it is the most relevant in OR. However, their model assumes that a packet can be indefinitely re-transmitted until it finally reaches the destination, and the system is solved using a different method.

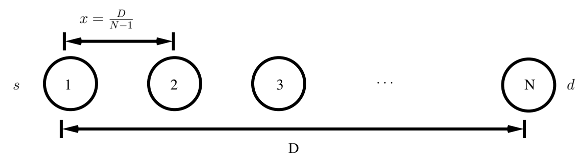


Fig. 1. Linear topology

The analytical model for OR based on Markov chains that we propose allows to evaluate the performance of OR in a given network scheme in a simple yet effective way. To the best of our knowledge, our model is the first one to consider a finite number of re-transmissions in the case where none of the next-hop nodes receive the packet. At the same time, our model provides a very intuitive analysis and allow for evaluating the impact of candidate list size as well as number of re-transmissions in any network topology.

III. Markov Model

In this section, we introduce our proposal to model Opportunistic Routing (OR) by a Discrete Time Markov Chain (DTMC). As we have mentioned in Section II, the research on OR topics has been extensively supported by simulation, it has not been so commonly supported by analytical models.

We can establish an analogy between a Markov chain and OR. Assuming perfect coordination between candidates at each packet transmission, only one candidate will forward the packet and the transmission will arrive at the next-hop following a certain probability. There are some proposals in the literature, such as three-way handshaking or RTS-CTS coordination [16], which can provide perfect coordination in OR. However, the focus of this work is the DTMC modeling to be applied to OR, so, to simplify the initial assessment, the coordination between candidates is left out of scope. In addition, for a candidate to receive a packet and to forward it, the source of the packet is not relevant. In the same way, in a Markov chain, at each step the system can be only in one state and the future states depend only on the current state, but not on the past ones. Furthermore, in OR, each candidate can receive a packet from the previous node with a certain probability and the transmission of the packet will stop when it reaches the destination or the packet is dropped by any nodes. As we can see, the properties of OR are very similar to a Markov chain with absorbing states; which are in this case the equivalent to the delivery or dropping of the packet. In the following sections, we show how an OR protocol can be modeled by a Markov chain.

A. Markov model for OR and linear topology

Consider a linear network topology shown in Figure 1 with N nodes numbered from 1 to N and equally spaced x meters, with s and d as the source and the destination, respectively. The candidate set from s to reach destination d, is $C_d^s = \{c_1, c_2, ..., c_{ncand}\}$. Note that the candidates are ordered according to their priority to be used as next-hop forwarders:

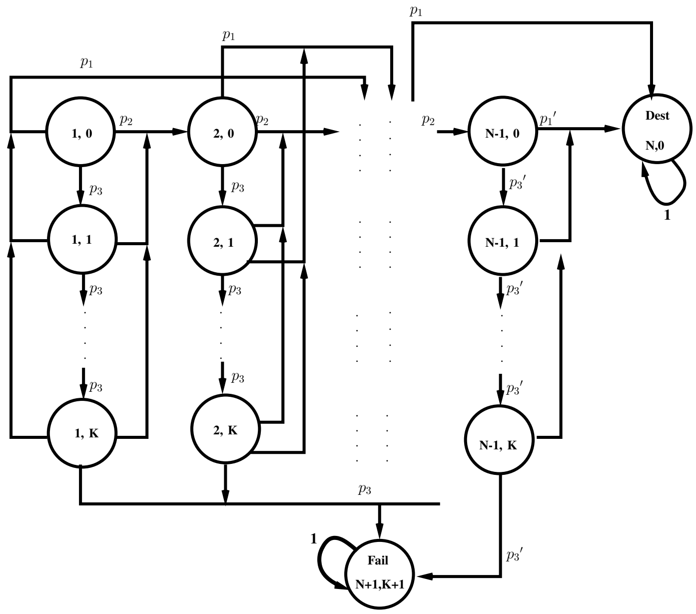


Fig. 2. Markov Model

$c_1 > c_2 > \cdots > c_{ncand}$. That is, candidate c_j will forward the packet if none of the higher priority candidates ($c_i, i < j$) receives the packet.

The source broadcasts a packet that will be forwarded by the highest priority candidate which received the packet, while the lower priority candidates will discard it. In other words, we have assumed that there is a perfect coordination between candidates. If the transmitted packet does not reach any of candidates, a node can re-transmit it at most $ReTx = K$ times. This process continues until the packet reaches the destination or until it is discarded by one of the intermediate nodes after K failed re-transmissions.

We can model the OR linear topology with a Discrete Time Markov Chain (DTMC) like the one depicted in Figure 2. For the sake of clarity, we have assumed that the number of candidates in each node is $ncand = 2$. Each state in our model represents a node and the number of re-transmissions being performed by that node to make the packet progress towards the destination. The state transitions in our Markov chain reflect how the packet progresses through the network. If the packet progresses towards the destination, it moves to the right in the Markov chain. On the contrary, if the packet needs to be re-transmitted, it will be moved to its respective downward state. Note that the obtained Markov model has two absorbing states (see Figure 2): state *Dest* corresponds to the successful delivery of the packet to the destination and *Fail* state corresponds to a dropped packet because the node has transmitted the packet K times.

Each state in our model is labeled with <ID, re-tx> where *ID* is the node identification and *re-tx* is the number of times that the packet is being re-transmitted by that node. Recall that each node is allowed to re-transmit a packet at most ReTx=K.

The first row of states, in the model presented in Figure 2, corresponds to the case in which nodes can deliver the packet to one of the candidates in the first transmission. The second row of states, ($<$ID, $1>$, ID $\in \{1, 2, \ldots N-1\}$) signifies that the first transmission failed and that the node needs to re-transmit the packet. This sequence of re-transmissions continues until a node has completed ReTx=K re-transmissions (as shown in the subsequent rows), and then the packet is dropped: i.e., it reaches the *Fail* absorbing state. If every node can deliver the packet to one of its candidates, the packet will eventually reach the other absorbing state (i.e. *Dest* state); this represents the successful delivery of the packet to the destination node. Note that there are no transitions between the states in the second and the lower rows (all rows except the first one), because when a re-transmission reaches one of the candidates, that candidate will continue forwarding the packet in its first transmission attempt.

To build up the transition probability matrix, we need to obtain the transition probabilities between states. Let us assume that $p(x)$ is the probability of reaching a node located at a distance x. As we mentioned above, with perfect coordination between candidates, the lowest priority candidate will forward the packet if the highest priority one does not receive the packet. Therefore, the probabilities corresponding to each transition between each state can be obtained as follows:

$$\begin{aligned} p_1 &= p(2x) \\ p_2 &= p(x) \times (1 - p_1) \\ p_3 &= 1 - (p_1 + p_2) \\ p_1' &= p(x) \\ p_3' &= 1 - p_1' \end{aligned} \tag{1}$$

where p_1 and p_2 are the probabilities that the highest priority candidate (c_1) and the second highest priority candidate (c_2) will forward the packet, respectively. Note that each node will re-transmit the packet, if none of the candidates (c_1 and c_2) receives the packet. This has been shown as p_3 in Equation 1. The probabilities p_1' and p_3' are the probabilities of reaching the destination in the last hop or having to re-transmit the packet, respectively.

Note that the Markov model presented in Figure 2 applies to a linear topology like the one shown in Figure 1 with ncand=2 candidates and ReTx=K. In the next section, we will show how we can apply our proposed model to any kind of network topology; this can be done regardless of the number of candidates and the candidate selection algorithm.

B. How to generalize and to solve the OR Markov chain

As we have showed in the previous section, there is a great similarity between OR and a DTMC; and, we have shown how we can model an OR linear topology with $ncand = 2$ candidates using a Markov chain. We can obtain a similar Markov chain model for $ncand \geq 3$. Furthermore, the model is extendable to any kind of network topology. The only input parameters needed to build the transition probability matrix are the candidates of each node, the delivery probabilities to reach them, and the maximum number of re-transmissions (ReTx). To obtain the input parameter for our model any candidate selection algorithm can be applied on the network topology to find the candidate set from source to the destination, and the candidate set is given to the model as one of its necessary inputs. As we can see, the obtained model can be applied with any number of candidates and candidate selection algorithms.

To solve the proposed Markov chain the first thing needed to be defined is the transition probability matrix and the dimension needed to build it. According to the definition of each state in Section III-A, in a network with N nodes and a maximum number of re-transmissions ReTx=K, the number of transient states in our model is equal to $(N-1) \times (K+1)$, in addition to the two absorbing states. Therefore, the transition probability matrix has a dimension equal to $[(N-1) \times (K+1) + 2] \times [(N-1) \times (K+1) + 2]$.

Recall that, each state in our model is labeled with <ID, re-tx>. Each transient state (all states except *Dest* and *Fail*) is mapped to an index in the transition probability matrix according to the node's ID and the number of re-transmissions so that *index*=$ID + (re-tx)\times$(N-1). For the absorbing states we consider the state $index = (N-1) \times (K+1) + 1$ as the state where the packet is dropped (*Fail*) and state $index = (N-1) \times (K+1) + 2$ as the destination state (*Dest*).

The formal definition of the transition probabilities of our model is completed by the set of equations in 3, in which the transition probabilities between all states are obtained. Note that $c_i(l), l \in \{1, 2, \ldots, ncand\}$, in Equation 3, is the candidate of node i with priority l. The first probability in Equation 3 is the probability of reaching the state $< c_i(1), 0 >$ from state $< i, k >$ (i $\in \{1, 2, \ldots, N-1\}$ and k $\in \{0, 1, \ldots, K\}$); and; this is equal to the probability of reaching the candidate $c_i(1)$ from node i. The probability of reaching the other candidates, but not the highest priority one, is shown in the second probability in Equation 3. The probability of performing a re-transmission for each state can be obtained by the third probability in Equation 3. Finally, the probability of remaining in each state with the exception of the *Dest* and *Fail* is equal to 0 (see the forth probability in Equation 3).

$$p^{i,j}_{i',j'} = \begin{cases} p(i, c_i(1)), & \begin{cases} 1 \le i \le N-1, i' = c_i(1) \\ 0 \le j \le ReTx, j' = 0 \end{cases} \\ p(i, c_i(k)) \prod_{l=1}^{k-1} (1 - p(i, c_i(l))), & \begin{cases} 1 \le i \le N-1, i' = c_i(k) \\ 0 \le j \le ReTx, j' = 0 \\ k = 2, \ldots, ncand \end{cases} \\ 1 - \sum_{l=1}^{ncand} p^{i,j}_{c_i(l),0}, & \begin{cases} 1 \le i \le N-1, i' = i \cup (N+1) \\ 0 \le j \le ReTx, j' = j+1 \end{cases} \\ 1, & \begin{cases} i = i' = N \text{ and } j = j' = 0 \\ or \\ i = i' = (N+1) \text{ and } j = j' = (K+1) \end{cases} \end{cases} \quad (3)$$

For our convenience, P can be expressed in its canonical form as:

$$P = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix} \quad (4)$$

where $Q_{(N-1)(K+1) \times (N-1)(K+1)}$ contains the transition probabilities between transient states, $R_{(N-1)(K+1) \times 2}$ contains the transition probabilities from transient states to the absorbing states and $I_{2 \times 2}$ denotes an identity matrix.

In order to build up the transition probability matrix, all we need is the candidate set of each node and the maximum number of re-transmissions K. We can then find out different metrics such as the probability of a successful transmission from a source to a destination using OR, the probability of a failed transmission and the expected number of transmissions needed to reach the destination. In the following text we explain how we can derive this information from matrix P.

The transition matrix P shows the probability for going from one state to every other state in one step. However, the destination may not be reachable from the source in only one step. To obtain the probability of reaching the destination from the source node, we need to find the probability of reaching the destination in h steps. This probability is given by the powers of P. Equation 5 shows the h-th power of P shows the probability of going from state i to other states after h steps.

$$P^h = \begin{bmatrix} Q^h & (I + Q + \ldots + Q^{h-1}) \times R \\ 0 & I \end{bmatrix} \quad (5)$$

In the stationary state $h \to \infty$, $(I + Q + Q^2 \ldots + Q^{h-1}) = (I - Q)^{-1}$. The matrix $N = (I - Q)^{-1}$ is known as the *fundamental matrix* of the Markov process. Note that $N \times R$ gives us the probability of being absorbed in every other transient state. Taking into account that we only consider the path from the source to the destination, the initial distribution of the states in our system is given by:

$$v = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \quad (6)$$

therefore, after h steps the probability of reaching any other state from the source is: $v \times P^h$, which corresponds to the first row of P^h. Particularly, $p_{1,1}$ in $N \times R$ presents the probability of dropping the packet which is transmitted by the source; and, the value of $p_{1,2}$ is the probability of a successful transmission from source to the destination.

Another very important metric in OR that can be easily derived from our Markov model is the expected number of transmissions from source to the destination. Let X be the random variable equal to the number of transitions from state i until the destination absorption state (*Dest* state). It is clear that the probability of transitioning from a transient state i to a transient state j in exactly $h-1$ steps is the $q_{i,j}$ entry of Q^{h-1} in matrix P^{h-1}. Therefore, the probability of transitioning from a non-absorbing state i to the absorbing state j, in h steps, can be obtained from Equation 7.

$$p[X = h] = Q^{h-1} \times R \quad (7)$$

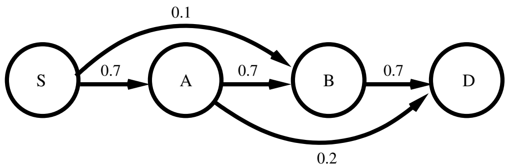


Fig. 3. Example of a topology

Note that $Q^{h-1} \times R$ is a matrix with dimensions $(N-1)(k+1) \times 2$. The expected value of X is the expected number of transmissions, which can be derived from Equation 8, where matrices are divided element by element.

$$E[X] = \frac{\sum_{h=1}^{\infty}(h \times p[X=h])}{p[X=1]+p[x=2]+\dots} = \frac{1 \times R + 2Q \times R + 3Q^2 \times R + 4Q^3 \times R + \dots}{R + Q \times R + Q^2 \times R + Q^3 \times R + \dots} = \frac{(I-Q)^{-2} \times R}{(I-Q)^{-1} \times R} = \frac{N^2 \times R}{N \times R} \quad (8)$$

C. An example of OR Markov model

We close the description of our Markov model by providing a simple example to show how we can use our model to evaluate the performance of a network using OR. To do so, we consider a network topology with four nodes in Figure 3. The numbers on each link represent the successful link delivery probability. Assume that each node can select at most 2 candidates and that the maximum number of re-transmissions in each node is equal to $K = 1$. The candidate set for node S to reach the destination D is $C_S^D = \{B, A\}$; here B is the highest priority candidate. Furthermore, the candidate set for A to the destination is $C_A^D = \{D, B\}$.

This topology and the candidate set in each node lead us to the Markov chain shown in Figure 4(a). For the sake of clarity, we have included in Figure 4(a) the index corresponding to every state in small round ovals next to each of state. These indexes are assigned also due to the positioning in the matrix.

In our example, the transition probability from the source to its second candidate (node A) is equal to $p_1 = 0.7 \times (1-0.1) = 0.63$. However, the source can reach its first candidate with a probability of 0.1. The source has to once again re-transmit the packet in case none of the candidates received the original transmission; it does so with probability $p_3 = 0.27$.

By building the matrix P and by applying the $(I-Q)^{-1} \times R$ and Equation 8, we can easily obtain the probability of reaching the destination and the expected number of transmissions that use OR to reach the destination D. We can see that the probability of reaching the destination state from the source is 0.81 and the the ExNT from source to the destination is 3.17.

Note that OR with only one candidate is a traditional routing which selects the shortest path from source to the destination. Our model can also be applied to traditional routing. Applying our model to traditional routing with path $S-A-B-D$, as shown in Figure 4(b), can result in a probability of reaching the destination equal to 0.75 with 3.69 as the expected number of transmissions needed to reach the destination. As we can see, OR outperforms traditional routing in terms of successful delivery for reaching the destination with less ExNT.

IV. Experimental analysis and validation

In this section, we show how our model can be used to evaluate the performance of OR in different scenarios. Note that, as we have mentioned in Section III, our Markov model can be applied on any kind of network topology and the candidate selection algorithm. The only ingredients needed in our model are the following: the candidate sets, $C_i^D = \{c_i(1), c_i(2), \dots, c_i(ncand)\}$; the delivery probability of reaching the candidates; and, the maximum number of re-transmissions, K, which is allowed in each node.

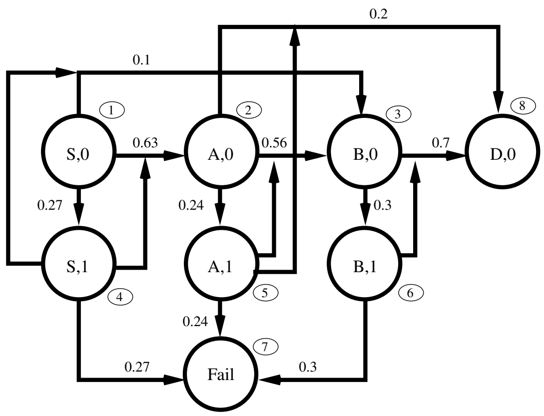
We have used the shadowing model with parameters $\beta = 2.7$ as the packet loss and $\sigma_{dB} = 6.0$ for standard deviation value in our numerical analysis (see [11] for more information). As the threshold for the probability to accept a link as an existing link between two nodes we have used $min.dp = 0.1$. That is, if the link probability between two nodes is greater than 0.1, we consider that link to be link in the network.

A. Numerical results

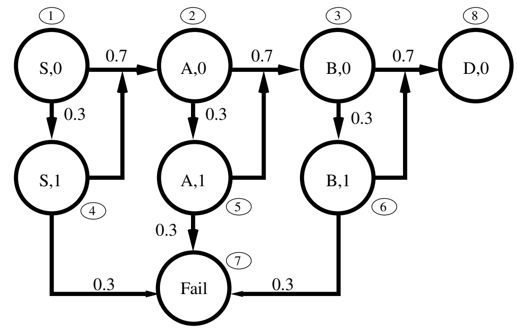
In this section, we show the numerical results obtained by applying our model on a linear topology. To show the effect of a different number of re-transmissions and candidates, we have used two different scenarios.

Recall that our Markov model can is applied on any kind of network topology and the candidate selection algorithm. Therefore, we have used a linear topology to enact a simple scenario to illustrate our results. We choose ExOR [3] to select the candidates in our experiments; this is because ExOR is a simple and the most well-known candidate selection algorithm. We have considered a linear topology with $N = 20$ nodes placed at a distance of $x = 100$ m (see Figure 1). Nodes $v_s = 1$ and $v_d = 20$ are considered as source and destination, respectively. In the following text, the notation OR_m^k refers to OR with $ReTx = k$ and $ncand = m$ as the maximum number of candidates for each node. When we measure the effect of these parameters (re-transmissions or number of candidates), the measured parameter is not shown in the notation.

1) The effect of a different number of re-transmissions in OR: For our first experiment, we have set the maximum number of candidates in each node as equal to 2 ($ncan = 2$). One of the important measurements in OR is the probability of reaching the destination for a different number of re-transmissions. Figure 5 depicts the results for the probability of reaching the destination (see state *Dest* in Figure 2), by varying number of re-transmissions for OR with ncand=2 (OR_2) and uni-path routing. In our comparison, we have added the results of traditional routing, which uses the ETX [17] of each link to find the shortest path from the source to the destination (see legend Uni-path in Figure 5). As we can see in Figure 5, allowing more re-transmissions results in a higher probability of arriving at the destination state; and, after 4 or 5 re-transmissions for OR_2 and uni-path, the probability is equal to 1. Furthermore, the results show that OR, even



(a) Markov model of OR in Figure 3



(b) Markov model of traditional routing in Figure 3

Fig. 4. Comparison of Markov chains for traditional and OR for the same network topology

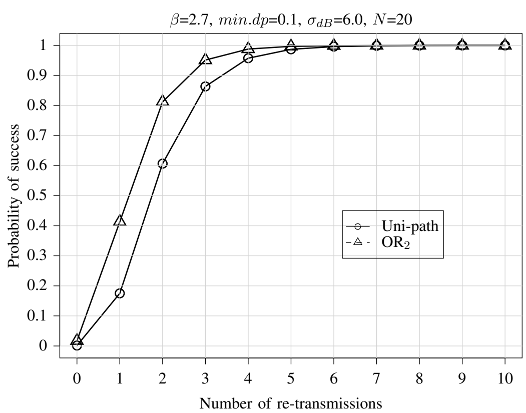


Fig. 5. Probability of success vs number of ReTx for N=20

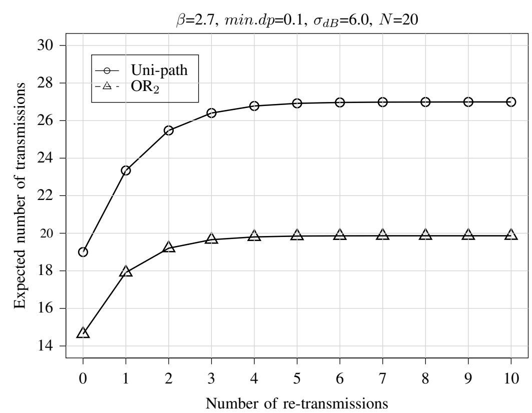


Fig. 6. Expected number of Transmissions vs Number of ReTx for N=20

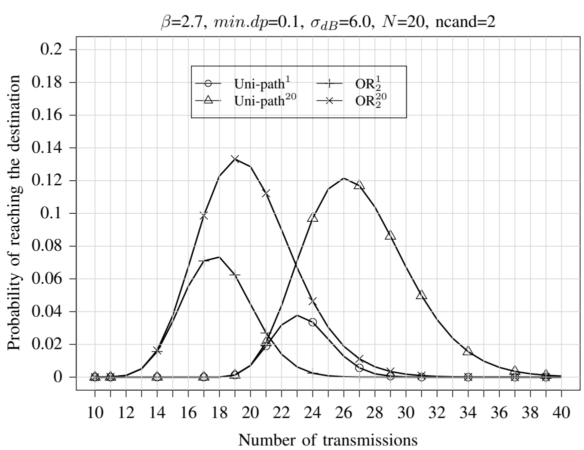


Fig. 7. Probability of number of transmissions, N=20, ncand=2.

with a number of candidates as small as 2, can increase the probability of reaching the destination compared to traditional routing. This comes from the opportunistic use of a set of nodes, instead of only one, which can act as the next-hop forwarder; this increases the chances to reach the destination. This conclusion has been obtained in [18] where the effect of the number of candidates in OR was investigated.

Another important and well-know metric in OR is the expected number of transmissions (ExNT) from source to the destination. Figure 6 shows the expected number of transmissions for the linear topology under study, varying the number of re-transmissions (ReTx=$0, 1, \ldots, 10$). As a first observation, in Figure 6, we can see that OR with only 2 candidates (OR_2) outperforms the uni-path routing in terms of the expected number of transmissions (ExNT). Furthermore, allowing more re-transmissions in each node increases the expected number of transmissions (ExNT), but after a certain number of re-transmissions the ExNT needed to reach the destination remains stable. For instance, in the case of OR_2, after $ReTx = 4$, the ExNT remains the same; on the other hand, for the traditional routing, the ExNT stops increasing after $ReTx = 6$. This comes from the fact that having more re-transmissions increases reliability; thus, the probability of a packet reaching the destination is increased. Therefore, the probability of being in state *Dest* and reaching the destination increases, by allowing a higher number of re-transmissions; this also increases the ExNT. However, increasing the number of re-transmissions over 4 for OR and over 6 for uni-path is not useful; it does not translate into gain in terms of the ExNT nor does it increase reliability. Considering the obtained results in Figure 5, we can see the difference between the probabilities of reaching the destination when the number of re-transmissions is greater than 3 and that the uni-path routing is about 1%; however, as we can see in Figure 6, OR can reach the destination in 26% less number of transmissions than uni-path routing.

Figure 7 shows a more detailed comparison of OR and uni-path routing by using our proposed Markov model to find the probability of the number of transmissions needed for a linear topology with N=20 nodes. The number of re-transmissions is set to 1 (OR^1 and Uni-Path^1); and, we also included the results for both routing mechanisms when the number of re-transmissions is big enough (ReTx=20) to satisfy the fact that the probability of being absorbed by the destination state is equal to 1 (legends OR^{20} and Uni-path^{20} in Figure 7).

First of all, Figure 7 confirms that by using OR, we can end at the destination state with a higher probability and a smaller number of transmissions. In other words, the obtained results in a network reveal that by using OR in the considered scenario results in a higher packet delivery ratio to the destination while less transmissions are needed than uni-path routing requires. As we can see, when there are enough re-transmissions to ensure that we end at the destination state (ReTx=20), by using OR (OR^{20}), 10% of the packets are delivered to the destination with 17 transmissions; on the other hand, the same percentage of packets (10%) reach the destination with 24 transmissions, in the case where uni-path routing (Uni-path^{20}) is used. The same situation will happen when we have a limited number of re-transmissions; ReTx=1. That is 7% of packets reach the destination with 17 transmissions, while in the uni-path routing about 4% of the packets are delivered to the destination with 23 transmissions. Note that the sum of every value obtained in each curve is equal to the corresponding point in Figure 5; e.g, the sum of the obtained values in the case of OR_2^1 is equal to 0.4, which is the same value at ReTx=1 for OR in Figure 5.

2) The effect of a different number of candidates in OR: As we have mentioned before, two of the main parameters which have an effect on the performance of an OR protocol are the number of re-transmissions of a packet in each node and the number of candidates. In this section we use our Markov model to investigate the effect of a different number of candidates on the performance of an OR approach. We have used the same linear topology, in this case with $N = 40$ nodes in the network, and a varying number of candidates ncand=$1, 2, 3, \ldots, 5$.

Figure 8 depicts the probability of reaching the destination with a different number of candidates ($ncand = 1, 2, \ldots, 5$) for different a number of re-transmissions; ReTx=$0, 1, 2, 3, 20$ (see legends in Figure 8). Recall that OR with $ncand = 1$ is equal to the uni-path routing approach. Therefore, the first point in Figure 8 represents uni-path or traditional routing for a different number of re-transmissions, while the other points correspond to OR.

The first observation for Figure 8 is the more candidates that are selected, the higher the probability of reaching the destination becomes. In addition, by having the same number of candidates and by increasing the number of re-transmissions (ReTx), the success probability is also considerably increased, at least for a number of candidates; ncan$<$ 3. Considering the results with no re-transmissions (OR^0), we can see that in order to increase the probability of success we need to include more candidates in the set. Note that with three candidates, the chances of reaching the destination are significantly higher than with one and two. The success probability in the case of 3 candidates with ReTx=2 is approximately 0.98. The difference with the probability in the case of 2 candidates and ReTx=$2, 3$ is very small; all probabilities are over 0.9. However, by having more candidates duplicated packets may be caused if there is imperfect coordination among the candidates.

Figure 9 depicts the ExNT for a different number of candidates with varying numbers of re-transmissions. As in Figure 5, we have added the results of uni-path routing. We can see that with any number of candidates OR outperforms the uni-path routing in terms of ExNT. Furthermore, the ExNT decreases as the number of candidates in the set increases. This is due to the greater chance of delivering the packet to the destination when there are more candidates; any candidate may receive the packet and forward it eventually; within a lower number of transmissions. As we can see, the increase in terms of ExNT for uni-path stops after approximately 9 re-transmissions. However, in the case of 2 candidates (OR_2), the ExNT does not change after 3 re-transmissions. This can also be explained by the results in Figure 8 that show the probability of success with a different number of re-transmissions when using different number of candidates in the set. When the number of re-transmissions is ReTx$\geq$ 3, the transmissions always end up in the absorbing state of the destination. Therefore, allowing more re-transmissions when the probability of being absorbed by the destination is close to 1 does not increase the ExNT from source to the destination.

Figure 10 shows additional results for the probability of transmissions for N=40 nodes with a different number of candidates. In the case of 2 candidates, we have shown the results for a different number of re-transmissions (ReTx=$0, 1$). Note that *any* in the legends in Figure 10 shows any number of re-transmissions. This is because in the case of a large number of candidates, e.g 3, the results for any number of re-transmissions are the same.

The probabilities for transmissions in Figure 10 for a different number of candidates show that by using a greater number of candidates in OR, a smaller number of transmissions is caused. For instance, in the case of OR_5^{any}, 24% of the packets are delivered to the destination with 14 transmissions; however for this number of transmissions when there are 4 candidates (OR_4^{any}), about 20% of the packets reach the destination. Comparing the obtained results of OR_2^0 and OR_2^1, we can see that with ReTx=0, about 4% of the packets reach the destination with 16 transmissions; on the other hand, for ReTx=1, about 8% of the packets are delivered to the destination with the same number of transmissions (16).

V. CONCLUSION

Opportunistic Routing (OR) presents a profitable routing mechanism that increases reliability in wireless networks. However, the selection of candidates has a great impact on

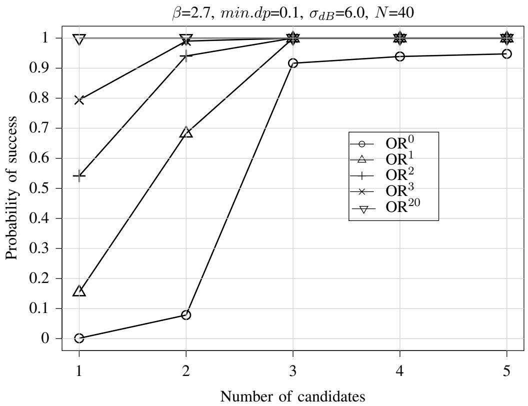


Fig. 8. Probability of success varying the number of candidates for N=40

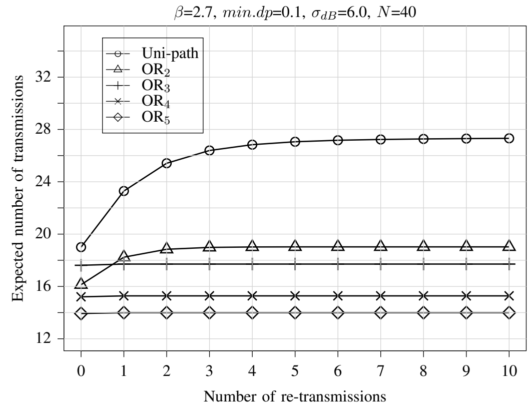


Fig. 9. Expected number of transmissions varying the number of re-transmissions for N=40

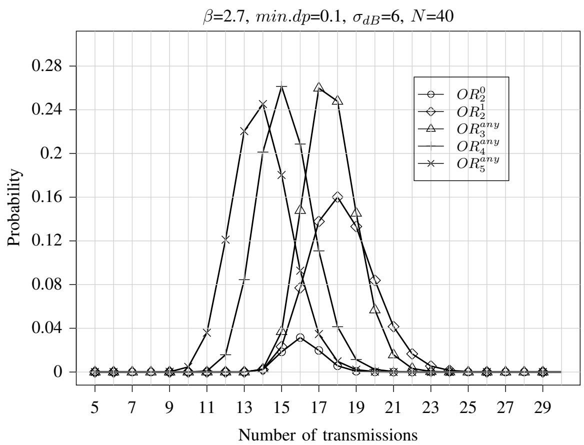


Fig. 10. Probability N=40, ReTx=5 different candidates

network performance. Most of the existing research focuses on this aspect from a simulation-based approach.

In this paper, we have proposed a Discrete Time Markov chain (DTMC) that enables the performance of OR to be evaluated in a very simple, fast and effective way. Our model is valid for any network topology and any candidate selection algorithm and can be evaluated through it. In addition, our model accounts the number of re-transmissions in the case none of the candidates receive the packet. To the best of our knowledge, our model is the first one that takes this issue into account. The proposed model takes into account the number of re-transmissions and is valid for any number of candidates, so both parameters can be easily evaluated when designing an OR protocol. As a result, the use of our model can provide guidelines to other researchers for evaluating the influence of the number of candidates, how this relates to the number of re-transmissions and how these two parameters together contribute to the successful delivery of the data packets.

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