# Opportunistic Beamforming for Finite Horizon Multicast

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***Abstract*—Wireless multicasting suffers from the problem that the transmit rate is usually determined by the receiver with the worst channel. Composite or adaptive beamforming allows using beamforming patterns that trade off antenna gains between receivers. A common solution for wireless multicast with beamforming is to select the pattern that maximizes the minimum rate among all receivers (for a given transmit power). However, when using opportunistic multicast to transmit a finite number of packets to all receivers – the finite horizon problem – this is no longer optimal. Instead, the optimum beamforming pattern depends on instantaneous channel conditions as well as the number of received packets at each receiver. We formulate the finite horizon multicast beamforming problem as a dynamic programming problem to obtain the optimal solution. We further design a heuristic that has sufficiently low complexity to be implementable in practice and show through extensive simulations that our algorithm significantly outperforms prior solutions.**

## I. Introduction

Wireless multicast is an efficient technique to disseminate multimedia data to groups of users. Using the broadcast nature of the wireless medium allows data to be served to multiple users simultaneously, but at the same time this constrains the transmit rate to the rate that can be supported by the receiver with the worst channel conditions.

To address this issue, a range of different mechanisms have been proposed. With opportunistic multicast scheduling (OMS), a base station (BS) exploits multiuser diversity and may opportunistically transmit to a subset of receivers that experience good channel conditions. This trades off multicast gain (achieved by transmitting to as many receivers as possible) versus multiuser diversity gain (achieved by transmitting to a subset of receivers that currently experience good channel conditions) [1]–[3]. Multiuser diversity gain can be exploited if receivers with a bad channel are likely to experience better channels in the future, and vice versa. It is particularly suitable for homogeneous scenarios where receivers' long-term average rates are similar, but instantaneous rates are highly variable. If, however there are some receivers that are clearly worse than the rest, the overall rates are still limited by those receivers. In this case exploiting multiuser diversity and multicast to all receivers are detrimental to performance. However, this also means that the transmit rate is limited by these worst receivers.

To overcome this problem, transmit beamforming can be used to adjust antenna gains to the different receivers. This allows improving the signal-to-noise ratios (SNR) of receivers with bad instantaneous channels at the expense of worsening those of receivers with better instantaneous channels. There are two main techniques for multi-user beamforming: (i) composite beamforming [4] and (ii) adaptive beamforming [5]. A composite beam is composed of multiple pre-determined single-lobe beam patterns. In contrast, adaptive beamforming calculates antenna weights directly based on the measured channels to the different receivers. While composite beamforming has lower complexity, adaptive beamforming may achieve better performance in multi-path rich environments. Similar to opportunistic multicast, the main challenge when designing multi-user beamforming mechanisms is the tradeoff between high gain beamforming to few receivers versus lower gain beamforming to a larger receiver set [4], [6], [7].

Most of the prior work in these areas aims at maximizing the rates of the receivers, which is optimal for the *infinite* horizon problem. In contrast, we analyze the more realistic *finite* horizon problem where the BS sends a block of a certain size to all receivers. In case there are multiple blocks to be sent, the BS starts transmitting packets for the next block only after all receivers have received the first data block. This is relevant for most practical scenarios where reliable delivery as well as delay constraints are a concern, as for example for multicast video streaming. Similar to prior work (e.g., [1]) we assume erasure coding of transmitted data, which is highly beneficial in wireless multicast scenarios and ensures that each packet received by a receiver is useful (with high probability).

This paper focuses on the finite horizon opportunistic multicast beamforming problem. In the infinite horizon problem, it is possible to exploit opportunistic gain very aggressively, since lagging receivers have an infinite amount of time to catch up. In contrast, in the finite horizon case the optimum decisions on when to exploit opportunistic gain and when to favor lagging users very much depend on the state of the receivers (i.e., the amount of data received thus far and therefore how close the receivers are to finishing). This substantially changes the formulation of the problem compared to the infinite horizon problem. In addition, the higher the number of users, the higher the multi-user diversity and therefore the potential for opportunistic gain. Exploiting opportunistic gain aggressively leads to higher average rates but at the same time may lead to some users finishing early, thus reducing the future total achievable throughput.

We first model the problem and obtain the optimum solution

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via dynamic programming. This allows us to study the impact of receiver state, instantaneous channel conditions, and average channels on the optimum receiver set to transmit to, and hence the optimum beamforming pattern. First, we study these tradeoffs in toy scenarios with two users. These insights allow us to design a low complexity heuristic algorithm that captures the main characteristics of the optimum solution and at the same time can run in real-time in the practical wireless scenarios. We perform a range of simulations for larger scenarios for homogeneous and heterogeneous receiver sets with realistic channels (Rayleigh fading). While the complexity of the dynamic programming solution prevents us from solving those larger problem instances optimally, we see that our proposed heuristic provides significantly better performance than solutions based on broadcasting or greedily maximizing rates. Note that both the broadcast and greedy mechanisms use beamforming and take the instantaneous channel conditions into account. The greedy mechanism is thus optimal for the infinite horizon case (or problem instances with very large block sizes) in homogeneous scenarios as presented in [8]. The broadcast mechanism makes use of beamforming to maximize the minimum rate and hence does not suffer from receivers with a bad channel severely limiting the transmit rate as in OMS. It corresponds to the solution in [7] that is optimal for scenarios with fixed channels but may be too conservative in case of variable channels. It is also optimal for scenarios with variable channels where the receiver set is very heterogeneous and one receiver has a significantly worse average channel than the other receivers.

The paper is structured as follows. A review of state-of-the-art for opportunistic multicast and multicast beamforming is given in Section II. In Section III, we model the finite horizon opportunistic multicast beamforming problem and provide an optimum solution based on dynamic programming. We design a low complexity heuristic *FH-OMB* in Section IV and analyze its performance in comparison to the optimum solution and other prior schemes in Section V. Section VI concludes the paper and provides an outlook on future work.

## II. Related Work

*Opportunistic Multicasting:* Opportunistic multicasting has been well studied for both the infinite horizon problem [1], [9]–[12] as well as the finite horizon problem [2], [3]. Among the first ideas to address the infinite horizon problem for homogeneous scenarios was to split the receivers into two groups according to their instantaneous channels and serve the group with the better channel quality. As the composition of the group changes from slot to slot, all users have equal chances to be served [9]. This work was extended in [1] by optimizing the selection ratio, i.e., the size of the receiver set to transmit to. As a single pre-computed selection ratio is not always optimal, [10] and [11] propose a dynamic user selection mechanism that depends on the instantaneous channel at each transmission.

The authors of [2] solve the user selection problem for the finite horizon case using extreme value theory to minimize completion time. However, the user selection is only based on the instantaneous channel but not on the user state (i.e., the amount of data received by users). In wireless systems with packet loss, this is suboptimal since users may have received a different number of packets. The problem is addressed in [3], where it is shown that the optimal solution for the finite horizon problem needs to take receiver state into account. The paper analyzes the trade off between multicasting gain and multiuser diversity and provides an optimal algorithm as well as a low complexity heuristic.

The main challenge of opportunistic multicasting is to cope efficiently with receivers with bad channel conditions. In this context, transmit beamforming can be used to balance the users' SNRs.

*Multicast Beamforming:* Multicast beamforming provides a trade off between multicast gain and beamforming gain. Beamforming to receivers with poor channel conditions improves the rate of these receivers (but at the same time lowers SNR at other receivers). The basic algorithm proposed in [6] first transmits omnidirectionally to the receivers that have a high SNR and then beamforms sequentially to the remaining weak receivers. Better performance can be achieved by selecting the beamforming vector that maximizes the minimum SNR among all multicast receivers [13], [14]. In [4], receivers are partitioned into groups that are scheduled sequentially, which may outperform mechanisms that always beamform to all receivers. The paper proposes two multicast beamforming mechanisms, one that splits power equally among all beams and one that allows for asymmetric power allocation. Both mechanisms use composite beamforming, where a multi-lobe beam pattern that serves multiple receivers is composed of multiple single-lobe beam patterns. Reference [7] improves upon this work and provides an optimal solution for the equal power split and two different heuristics for the (NP-hard) asymmetric power allocation mechanism. Both [4] and [7] consider the finite horizon problem but do not take channel variations and opportunistic scheduling into account.

The same problem is addressed in [5] using adaptive beamforming rather than composite beamforming. Adaptive beamforming may provide better antenna gains than composite beamforming, in particular in multipath environments, but at the same time determining the optimum beamforming pattern is more complex.

*Opportunistic Multicast Beamforming:* There is very little existing work that jointly takes opportunistic multicast scheduling and multicast beamforming into account. [8] provides a theoretical analysis of the optimum user selection ratio for opportunistic multicast beamforming using extreme value theory. Once the user group is determined, the optimal beamforming pattern is the one that maximizes the minimum SNR among the users that are served. The algorithm is designed for independent and identically distributed users (i.e., homogeneous scenarios) for the infinite horizon multicast problem. We show that this approach is not suitable for the finite horizon multicast problem, especially for heterogeneous user distribution.

Our paper differs from prior work in that it addresses *finite* horizon opportunistic multicast beamforming in homogeneous and heterogeneous scenarios and explicitly takes into account receiver state (i.e., the amount of data already received).

## III. System Model

We consider a wireless network with a single BS (or access point) and a set $T$ of multicast receivers, with $|T| = N$. We assume the channels between BS and the receivers are independent discrete memoryless channels.[1] Let $\mathcal{G}$ denote the set of all possible vector channels from the BS to the receivers. The probability that at a given time instant the channel vector $\mathbf{C}$ has channel gains $\mathbf{g} \in \mathcal{G}$ is given by $P(\mathbf{C} = \mathbf{g})$. Let $C_i$, $g_i$, and $\mathcal{G}_i$ denote the corresponding channel instance, gain, and set of possible channels for receiver $i$. As is common for opportunistic scheduling, we assume that the BS has perfect knowledge of the current channel instance, but for any future channel instances only the channel distribution is known.

The BS uses composite beamforming with $K$ antenna elements. These generate $K$ antenna patterns that are optimized to produce one strong single-lobe beam that covers a sector of approximately $\frac{360^0}{K}$ and that together cover the whole azimuth of $360^0$. A composite beam is a multi-lobe beam pattern composed of several single-lobe beams that are transmitted simultaneously [4]. Each single-lobe beam $k$ has a certain beam weight $\alpha_k$. This weight corresponds to the fraction of the total transmit power allocated to that beam, and thus determines the SNR at the receivers covered by the beams. To ensure that the total radiated power remains unchanged, we have the constraint $\sum_k \alpha_k = 1$. Let $k_i^*$ be the strongest single-lobe beam that covers receiver $i$ and let $\gamma_{g_i}^{i,SLB}$ denote the SNR at that receiver when using that single-lobe beam when the channel gain is $g_i$. Then the SNR of that receiver for a multi-lobe beam is

$$\gamma_{g_i}^{i} = \alpha_{k_i^*} \gamma_{g_i}^{i,SLB} .$$

We consider a time-slotted model. In each time slot the BS transmits data to the receivers using a certain modulation and coding scheme (MCS) and beamforming pattern. For MCS $m \in M$, the number of bits transmitted in a slot is $R_m$ and the corresponding packet reception probability for an SNR of $\gamma$ is $p_m(\gamma)$. Note that we assume that receiver $i$ will only be served when a multi-lobe beam is used with $\alpha_{k_i^*} \neq 0$.

### A. Problem Formulation

The BS has a block of data of size $B$ (in bits) to transmit to all receivers. An erasure code is applied to the data before transmission, so that each data packet is useful for each receiver that receives it, as long as that receiver has obtained less than $B$ bits so far.

The optimization problem is thus for the BS to select at each time slot the multi-lobe beam pattern with corresponding weights as well as the MCS that minimizes the expected completion time. Optimal choice of beam pattern and MCS depend on the current instantaneous channel, the probability distributions of the channels, and the amount of data received by the receivers so far.

When beamforming to a subset of receivers $T' \subseteq T$, the highest expected rate to those receivers is obtained by selecting beam weights $\alpha_k^*$ that maximize the minimum SNR at the receivers. Let $T'_k = \{i \in T' : k_i^* = k\} \subseteq T'$ be the subset of receivers served by beam $k$. The minimum SNR of receivers in $T'_k$ for a single-lobe beam pattern and a given channel $\mathbf{g}$ is

$$\gamma_{\mathbf{g}}^{SLB}(T'_k) = \min_{i \in T'_k} \gamma_{g_i}^{i,SLB} \tag{1}$$

and, as shown in [7], the optimum weights for the multi-lobe beam pattern are thus given by

$$\alpha_k^* = \begin{cases} \left( \gamma_{\mathbf{g}}^{SLB}(T'_k) \sum_{j=1, T'_j \neq \emptyset}^{K} \frac{1}{\gamma_{\mathbf{g}}^{SLB}(T'_j)} \right)^{-1}, & \text{if } T'_k \neq \emptyset \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

This results in the same minimum SNRs for all lobes of the multi-lobe beam. Hence, all receivers in $T'$ are served with the MCS that provides the highest expected rate

$$m^* = \arg\max_m R_m p_m(\alpha_k^* \gamma_{\mathbf{g}}^{SLB}(T'_k)) . \tag{3}$$

Thus, rather than optimizing over all possible beam weights, it is sufficient to optimize over all possible subsets of receivers. (Note that the algorithm in [7] always serves all receivers associated with a given beam, while this is no longer optimal for opportunistic multicast. Consider a scenario where all receivers are located in the same beam. This is the conventional OMS scenario for which it is well known that broadcasting to all users is not always optimal [1].)

### B. Dynamic Programming Solution

With this we can formulate the problem as a stochastic shortest path problem and solve it through dynamic programming [15]. The state is given by the amount of data received by the receivers so far $\mathbf{s} = [s_1 \ldots s_N], 0 \leq s_i \leq B$ and we denote the state space by $\mathcal{S}$.[2] As all time slots have the same duration, the cost per slot is 1.

When multicasting to a subset $T'$ of receivers with an instantaneous channel of $\mathbf{g}$, the transition probability from state $\mathbf{s}$ to state $\mathbf{s}'$ is

$$\rho_{\mathbf{g}}^{T'}(\mathbf{s}, \mathbf{s}') = \sum_{\substack{\min(\mathbf{s}+R_{m^*}\mathbf{e}, \mathbf{B}) = \mathbf{s}' \\ \mathbf{e} \in \mathcal{E}}} \left( \prod_{i=1}^{N} p_{m^*}(\gamma_{g_i}^{i})^{e_i} (1 - p_{m^*}(\gamma_{g_i}^{i}))^{1-e_i} \right) \tag{4}$$

where the vector minimization above is element-wise. $\mathcal{E} = \left\{ \mathbf{e} \in \{0,1\}^N \right\}$ is the set of binary vectors of size $N$ and $e_i$ is the $i$th element of $\mathbf{e}$, indicating whether receiver $i$ received the packet or not. The MCS $m^*$ is calculated according to Equation (3). Equation (4) takes into account all combinations

[1] Note that our heuristic works for continuous channels and we provide simulation results for Rayleigh fading channels in Section V.

[2] Given that there is a discrete set of rates $R_m$, many states cannot be reached and we remove these states from the state space to speed up the computation.

of which receivers will receive the packet and ensures that the state of receivers with $s_i = B$ does not change.

A policy $\mu_{\mathbf{s}} : \mathcal{G} \mapsto \bigcup_{T' \subseteq T} T'$ specifies the best subset of receivers to transmit to for any instantaneous channel $\mathbf{g}$ when in state $\mathbf{s}$. Let $\mathcal{M}$ bet the set of all possible mappings. Since the probability of terminating after a finite number of steps is positive, we can use Bellman's equation to find the optimal policy

$$\mu_{\mathbf{s}}^* = \arg\min_{\mu_{\mathbf{s}} \in \mathcal{M}} \left( \sum_{\mathbf{g} \in \mathcal{G}} P(\mathbf{C} = \mathbf{g}) \sum_{\mathbf{s}' \in \mathcal{S}} \rho_{\mathbf{g}}^{\mu_{\mathbf{s}}(\mathbf{g})}(\mathbf{s}, \mathbf{s}') D^*(\mathbf{s}') \right). \tag{5}$$

The corresponding optimal expected completion time is

$$D^*(\mathbf{s}) = \min_{\mu_{\mathbf{s}} \in \mathcal{M}} \left( \sum_{\mathbf{g} \in \mathcal{G}} P(\mathbf{C} = \mathbf{g}) \sum_{\mathbf{s}' \in \mathcal{S}} \rho_{\mathbf{g}}^{\mu_{\mathbf{s}}(\mathbf{g})}(\mathbf{s}, \mathbf{s}') D^*(\mathbf{s}') \right). \tag{6}$$

Given that the state space is finite we can solve the dynamic program through value iteration, starting from the final state $\mathbf{s}^B$. This optimization problem is hard and even a much simpler version of it with fixed channels (i.e., no opportunistic scheduling), as well as guaranteed packet delivery without errors is NP-hard [7].

The dynamic program has double exponential complexity. The state space has size $B^N$ and for each state there are $2^{N^{|\mathcal{G}|}}$ policies that map each of the channel states in $\mathcal{G}$ to one of the $2^N$ possible multi-lobe patterns. Also $|\mathcal{G}|$ itself is exponential in $N$. Clearly, the dynamic program can only be solved for very small problem instances. For this reason, in the next section we design a lower complexity heuristic.

## IV. Heuristic Algorithm

Our Finite-Horizon Opportunistic Multicast Beamforming (*FH-OMB*) heuristic has two main parts: 1) given the current instantaneous channel, computing the next states the system could move to using the different multi-beam lobes that correspond to multicasting to the different subsets of receivers, and 2) estimating the expected completion times from those new states. The decision taken by the heuristic is then to beamform to the subset of receivers that results in moving to the state with the lowest expected completion time.

### A. Instantaneous Beamforming Decision

Let the current state be $\mathbf{s}$ and the current instantaneous channel be $\mathbf{g}$. Assume the estimated completion times $D(\mathbf{s}')$ for all future states are known. When beamforming to $T' \subseteq T$ we can calculate $\gamma_{\mathbf{g}}^{SLB}(T_k')$, $\alpha_k^*$, and the resulting optimum MCS $m^*$ using Equations (1)–(3). The expected future state $\mathbf{s}'(T')$ is given by

$$s_i'(T') = \min\left(s_i + R_{m^*} p_{m^*}(\gamma_{g_i}^i), B\right) \;\; \forall i$$

and the optimum subset of receivers $T^*$ to beamform to is thus

$$T^* = \arg\min_{T' \subseteq T} D(\mathbf{s}'(T')). \tag{7}$$

In contrast to the dynamic programming formulation we compute expected average future state rather than looking at all combinations of possible future states based on packet loss events. Note that this still requires minimization over a number of completion times that is exponential in the number of receivers, which can be done exhaustively for small receivers sets.

For larger receiver sets, we cluster receivers according to their state $s_i$ and relative quality of the instantaneous channel. The rate receiver $i$ would obtain with the current channel $g_i$ for a single-lobe pattern is $R(i) = \max_m R_m p_m(\gamma_{g_i}^{i,SLB})$, and the average rate that is obtained under all possible channels is

$$\bar{R}(i) = \sum_{g_i \in \mathcal{G}_i} P(C_i = g_i) \max_m \left(R_m p_m(\gamma_{g_i}^{i,SLB})\right). \tag{8}$$

The relative channel quality is $R(i)/\bar{R}(i)$. Let $0 = \xi_1 < \xi_2 < ... < \xi_U = B$ be a set of state thresholds and $0 = \theta_1 < \theta_2 < ... < \theta_V = \infty$ be a set of relative channel quality thresholds. We then group all receivers with

$$T_{uv} = \left\{ i \in T : \xi_u \leq s_i < \xi_{u+1}, \theta_v \leq R(i)/\bar{R}(i) < \theta_{v+1} \right\}$$

where the total number of groups is $UV$. In Equation (7) only optimize over subsets $T' \subseteq T$ that include whole receiver groups (i.e., if one of the receivers in a group is included, the whole group must be included). We set the thresholds so that the receivers are distributed relatively evenly among the groups. In order to further reduce the number of schedules, a group can only be scheduled if all groups that have higher channel state (i.e., better relative channel quality) and at the same time have lower receiver state as well are scheduled. With this, the maximum number of combinations of schedules which indicates the complexity of the *FH-OMB* is

$$O\left(\frac{(V+U-1)!}{U!(V-1)!}\right). \tag{9}$$

The number of beamforming patterns is fixed for fixed value of $U$ and $V$.[3]

### B. Estimating the Expected Completion Time

The main complexity of the dynamic programming solution lies in the calculation of the expected completion time. Hence, this is what the heuristic primarily addresses. As only the instantaneous channel is known at the BS, we base the expected completion time of a future state on the average channel of the receivers. Due to the shape of the rate function, simply averaging the channel would overestimate the receive rate. Hence we first calculate the average single-lobe rate of receiver $i$, $\bar{R}(i)$, as given by Equation (8) and then set the receiver's average SNR $\bar{\gamma}_g^{i,SLB}$ such that

$$\max_m \left(R_m p_m(\bar{\gamma}_g^{i,SLB}))\right) = \bar{R}(i).$$

For *fixed* SNRs and a continuous rate function, according to [7] the maximum rate when multicasting to a receiver set

[3]From the simulations we find that a reasonably low value for $U$ and $V$ (i.e., $U = V = 4$) suffices in practice, leading to a fixed number of subsets to consider for the optimization.

is obtained for a multi-lobe beam pattern that encompasses the whole receiver set. Analogous to Equations (1) and (2), for a receiver subset $T'$ we can derive $\bar{\gamma}_{\mathbf{g}}^{SLB}(T'_k)$ as well as $\bar{\alpha}_k^*$ based on the average SNRs $\bar{\gamma}_g^{i,SLB}$ calculated above. The corresponding hypothetical average rate is given by

$$\bar{R}(T') = \bar{R}(T'_k) = \max_m R_m p_m(\bar{\alpha}_k^* \bar{\gamma}_{\mathbf{g}}^{SLB}(T'_k)) \,. \tag{10}$$

We have $\bar{R}(T') = \bar{R}(T'_k)$ for any non-empty lobe $k$, since all lobes have the same minimum rate.

With this, we can now approximate the expected completion time as follows. For a given state $\mathbf{s}$, let $T'_1 = \{i \in T : s'_i < B\}$ be the set of receivers that still require further packets and let $s'^{(1)}_{\text{max}} = \max_{i \in T'_1} s'_i$ be the state of the receiver(s) closest to completing. When multicasting to this receiver set at rate $\bar{R}(T'_1)$ given by Equation (10), one or more of the receivers would complete after a time $\tau_1 = (B - s'^{(1)}_{\text{max}})/\bar{R}(T'_1)$. Determine the set of remaining receivers $T'_2 = \{i \in T'_1 : s'_i < s'^{(1)}_{\text{max}}\}$ and set $s'^{(2)}_{\text{max}} = \max_{i \in T'_2} s'_i$ to calculate $\tau_2$, etc. In general,

$$\tau_j = (B - s'^{(j)}_{\text{max}})/\bar{R}(T'_j) \,. \tag{11}$$

In other words, the estimation algorithm proceeds diagonally through the state space until hitting a boundary with $s_i = B$ for one of the dimensions, then proceeds diagonally along that boundary until hitting the next one, and so on, until reaching the final state. The algorithm terminates after at most $N$ steps. The expected completion time is given by

$$D(\mathbf{s}') = \sum_j \tau_j \,. \tag{12}$$

*Accounting for opportunistic gain:* When determining $\tau_j$ above, we assume that receivers in $T'_1$ are served first, then receivers in $T'_2$, etc. This ignores that receiver sets will be selected based (also) on their instantaneous channels. As a consequence, $\bar{R}(T')$ is a conservative estimate of the actual rate at which this receiver group is served, since they are more likely to be served when their channel is good. We refine Equation (11) to take into account opportunistic gain as follows. We assume that receivers in groups $T'_1$ and $T'_2$ are served during $\tau_1 + \tau_2$. If the channels of the receivers in $T'_1 \setminus T'_2$ are good, group $T'_1$ will be served, otherwise group $T'_2$ will be served. Hence, receivers in $T'_1$ see better average channels (since some of the beam weight $\alpha$ that was required for receivers in $T'_1 \setminus T'_2$ can now be used for other beams) whereas there is no change for receivers in $T'_2$. We remove the worst fraction $\tau_2/(\tau_1 + \tau_2)$ of channel combinations of the receivers in $T'_1 \setminus T'_2$ and update their average channels accordingly. We then recompute Equations (10) and (11) and obtain a new $\tau'_1$. Similarly, the calculation of $\tau'_2$ is based on receiver groups $T'_2$ and $T'_3$, and so on. The completion time is then calculated as $D(\mathbf{s}') = \sum_j \tau'_j$. Note that this is still a conservative estimate of the opportunistic gain.

*Example and discussion:*

To provide an intuition for the completion time estimation, we discuss an example for a two-receiver case in Fig. 1. In a

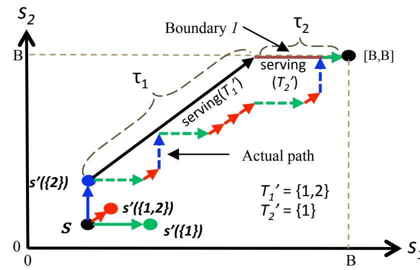


Fig. 1. Completion time estimation with the FH-OMB heuristic.

two-user scenario, there are only three possible beamforming patterns serving receiver sets $\{1\}$, $\{2\}$, or both $\{1, 2\}$. For $\{1\}$ and $\{2\}$, single-lobe beamforming patterns with maximum array gain to the respective receiver are used, whereas for $\{1, 2\}$ the multi-lobe beam that equalizes the SNRs of the receivers is chosen. In the latter case, both receivers are served at the same rate and have the same packet loss probability. For each of the average future states $s'(\{1\})$, $s'(\{2\})$, and $s'(\{1, 2\})$ we compute the expected completion time. Consider, for example, $s'(\{2\})$. Since both users have not yet finished, we calculate the number of time slots $\tau_1$ required for the first receiver to have $B$ bits. In the example this is receiver 2. We then compute $\tau_2$ required for the second user to complete, based on the single-lobe pattern to that user only. Using $\tau_2$, we can recompute the first segment to obtain $\tau'_1$ that partially accounts for opportunistic gain. $\tau'_2 = \tau_2$ since there is no opportunistic gain for a single receiver. The actual path that is taken (shown with dotted lines) depends on the actual instantaneous channel conditions at future states and is generally shorter than the sum of the estimated path segments.

An important observation is that determining the exact completion time is not important. What is important is to have approximately the right relative differences among completion times of nearby states (in this case $s'(\{1\})$, $s'(\{2\})$, and $s'(\{1, 2\})$), such that the right *instantaneous* beamforming decisions are taken. As a consequence, it is possible to use average channels instead of all possible channel combinations, without incurring a substantial drop in performance.

## V. Simulation Results

In this section, we present simulation results to analyze the performance of the algorithms. We first investigate a simple scenario with two receivers and a two-state channel to compare the optimal dynamic programming solution (*Dyn-Prog*) and the finite horizon opportunistic multicast beamforming heuristic (*FH-OMB*) and gain insights into the optimum strategy and fundamental tradeoffs. We then investigate more realistic scenarios with multi-path Rayleigh fading channels, larger number of receivers, and larger block sizes. For these, we do not provide dynamic programming results as the run time is prohibitive due to the algorithm's complexity. The multi-path Rayleigh fading channel corresponds to the ITU Pedestrian B path loss model in [16]. For all the scenarios, we use a subset of 13 MCSs given in the LTE specification for the 20MHz LTE downlink model (with modulation schemes QPSK, 16-QAM, and 64-QAM, and code rates from $0.1885$ to $0.9258$). The

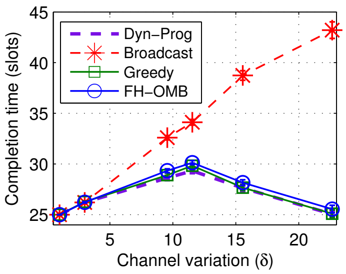


Fig. 2. Completion time in a homogeneous scenario with increasing channel variability.

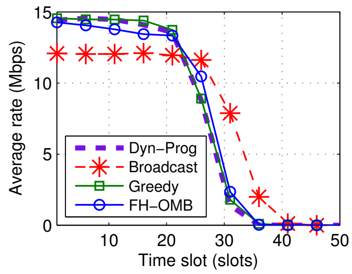


Fig. 3. Average throughput over time for a homogeneous scenario with channel variability $\sigma = 11.5$dB.

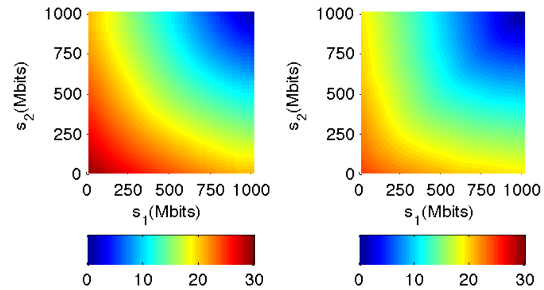


Fig. 4. Expected completion time for *Dyn-Prog* (left) and *FH-OMB* (right) for $\sigma = 11.5$dB.

corresponding transmit rates range from 5Mbps to 95Mbps. A time slot has a duration of 1ms. The main performance metric is completion time, i.e., the number of time slots needed for all receivers to receive $B$ kbits.

We compare the performance of *Dyn-Prog* and the *FH-OMB* heuristic with two alternative mechanisms:

*1) Broadcast Algorithm: Broadcast* uses a multi-lobe beam pattern that covers all receivers $i$ with $s_i < B$, maximizes the minimum SNR across all lobes, and serves the receivers with the optimum MCS $m^*$ for that SNR as given in Equations (1)–(3). This scheme is presented in [7] and it is shown to be optimal for constant channels with fixed SNR.

*2) Greedy Algorithm:* For *Greedy*, we sort the receivers with $s_i < B$ according to their instantaneous channel quality, given by the single-lobe SNR $\gamma_{g_i}^{i,SLB}$. Let $T_1$ be the receiver set that includes the receiver with the best channel (that hasn't finished yet), $T_2$ be the set of the two receivers with the two best channels, etc. The algorithm then determines the receiver set to beamform to as

$$T^* = \arg\max_{T_j} \sum_{i \in T_j} R_{m^*} p_{m^*}(\gamma_{g_i}^{i,SLB}) \,.$$

The optimum receiver set is the one with the highest overall sum rate for all receivers that have not yet finished. This algorithm corresponds to the one proposed in [8] and works well for homogeneous receiver sets.

### A. Simple Scenario

In this section, we present the results for a simple scenario with $N = 2$ receivers and block size $B = 1000$kbits. Each receiver $i$ has two possible instantaneous channels ($g_i = \{H_i, L_i\}$), such that $\mathcal{G} = \{H_1H_2, H_1L_2, L_1H_2, L_1L_2\}$ with $P(C_i = H_i) = P(C_i = L_i) = 0.5\,\forall i$. We analyze a homogeneous scenario and a heterogeneous scenario.

*1) Homogeneous Scenario:* In this scenario receivers have the same set of channels ($H = H_1 = H_2, L = L_1 = L_2$). We investigate the impact of channel variability, $\sigma = \gamma_H^{SLB} - \gamma_L^{SLB}$, i.e., the difference between the high gain channel and the low gain one. (For example, the left most point of Fig. 2 has $\gamma_H^{SLB} = 10$dB, $\gamma_L^{SLB} = 9$dB, $\sigma = 1$dB and the right most point has $\gamma_H^{SLB} = 18$dB, $\gamma_L^{SLB} = -4.7$dB, $\sigma = 22.7$dB). $\gamma_H^{SLB}$ and $\gamma_L^{SLB}$ values are chosen such that with single-lobe beamforming the receivers would achieve the same average rate and hence we can compare relative rate changes as the variability increases.

As shown in Fig. 2, both *Greedy* and the *FH-OMB* heuristic perform almost as good as the optimal *Dyn-Prog*. As both receivers have the same channel distribution, differences in receiver state are likely to cancel out over time and maximizing the instantaneous sum rate as *Greedy* does is a good strategy. Only when one receiver is close to finishing and the other receiver is lagging further behind may it be beneficial to favor the lagging receiver instead. Note that the graph also shows 95% confidence intervals but due to the large number of simulation runs they are very small.

For small channel variability ($\sigma < 3$dB), the maximum sum rate is achieved by serving both receivers for any of the channel combinations, hence *Broadcast* and *Greedy* have the same performance. Once the channel variability is increased beyond this point, beamforming only to the receiver with a good channel when the other receiver has a bad channel ($H_1L_2, L_1H_2$) provides higher throughput than beamforming to both receivers. Hence, *Broadcast* is unnecessarily conservative by always serving both receivers and its completion time increases substantially as the channels become more variable.

Since in such a homogeneous scenario maximizing sum throughput is almost always the right strategy, *Greedy* even slightly outperforms *FH-OMB* for higher channel variability. Due to this variability, receiver states may differ enough so that *FH-OMB*'s conservative completion time estimate prevents it from opportunistically exploiting good channels as aggressively as *Greedy*. This can be seen in more detail in Fig. 3, which shows average system throughput per time slot (averaged over all simulation runs and over both receivers, where receivers that finished have 0 throughput) for the scenario with channel variability $\sigma = 11.5$dB. Throughput of *FH-OMB* starts out the same as that of *Dyn-Prog* and *Greedy*, but drops off slightly once receiver states becomes more heterogeneous and one receiver is close to finishing. Fig. 4 shows the completion time estimates for the dynamic programming algorithm (left) and the *FH-OMB* heuristic (right) for the same scenario (i.e., $\sigma = 11.5$dB). *FH-OMB*'s completion time estimate based on average channels underestimates completion time when the channel is more variable, but the relative differences in estimated completion time for the different states for the two algorithms are very similar. *FH-OMB*'s completion time estimation algorithm thus leads to the right beam-forming decisions in most cases. The performance gap is due to

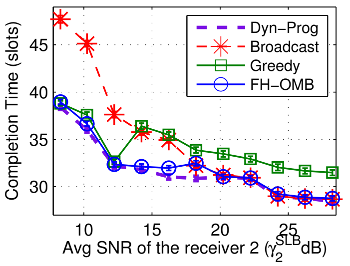


Fig. 5. Completion time in a heterogeneous scenario with increasing average SNR of the better receiver $\bar{\gamma}_2^{SLB}$ (i.e., receiver 2).

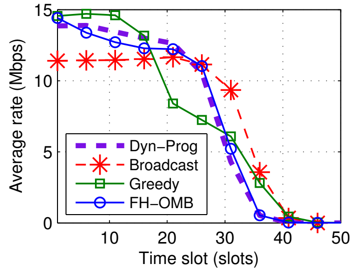


Fig. 6. Average throughput over time for a heterogeneous scenario $\bar{\gamma}_2^{SLB} = 14.2$dB.

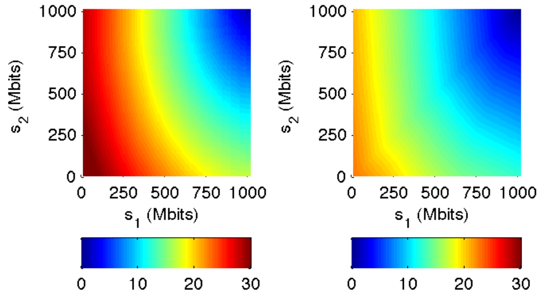


Fig. 7. Expected completion time for *Dyn-Prog* (left) and *FH-OMB* (right) for $\bar{\gamma}_2^{SLB} = 14.2$dB.

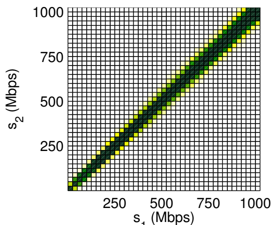


Fig. 8. State space visits for *Broadcast* at $\bar{\gamma}_2^{SLB} = 14.2$dB.

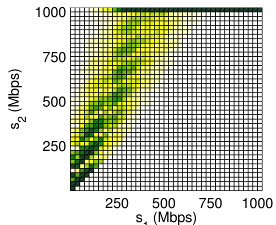


Fig. 9. State space visits for *Greedy* at $\bar{\gamma}_2^{SLB} = 14.2$dB.

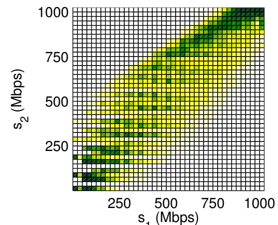


Fig. 10. State space visits for *Dyn-Prog* at $\bar{\gamma}_2^{SLB} = 14.2$dB.

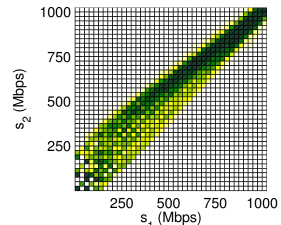


Fig. 11. State space visits for *FH-OMB* at $\bar{\gamma}_2^{SLB} = 14.2$dB.

the fact that *FH-OMB*'s completion time estimate is slightly less "round" than the true estimate, making it appear more beneficial to stay close to the diagonal where both receivers have the same state.

It is interesting to note that the completion time increases for 1dB $\leq \sigma \leq$ 11.5dB and then decreases again. When channel variation is low, both receivers are likely to finish at approximately the same time. The higher $\sigma$ , the more likely it becomes that one receiver finishes earlier than the other, which increases completion time given by the maximum of the individual completion times. When increasing $\sigma$ even further, completion times reduce since with a good channel, only very few time slots are needed to complete. There is a significant probability that one of the receivers will finish very early, and the system can then serve the remaining receiver at a higher rate with the corresponding single-lobe beam.

*2) Heterogeneous Scenario:* For the heterogeneous scenario, we fix the $\gamma_{H_1}^{SLB} = 11$dB and $\gamma_{L_1}^{SLB} = -1.4$dB of the first receiver. For the second receiver, we vary $\gamma_{H_2}^{SLB}$ between 11dB and 31dB and $\gamma_{L_2}^{SLB}$ between $-1.4$dB and 18.6dB, so that the two receivers become more and more heterogeneous as the channel values for the second receiver increase.

As the $\gamma_{H_2}^{SLB}$ and $\gamma_{L_2}^{SLB}$ increase, completion time decreases for all algorithms. *Greedy* performs close to optimal for the first three data points where receivers are sufficiently homogeneous and the optimum strategy is to beamform to the receiver with high channel gain when one receiver has high channel gain and the other receiver has low channel gain. Here, *Broadcast* is again too conservative. The jump in *Greedy*'s completion for the next data point is due to the fact that from this point on the good channel of the better receiver is so good that *Greedy* favors the receiver exclusively in that case and only serves both receivers when the good receiver has a low channel gain. In contrast, *Broadcast*'s strategy to balance the rates and forego opportunistic gain becomes closer and closer to optimal as the scenario becomes more heterogeneous and from an average SNR of $\bar{\gamma}_2^{SLB} \geq 17$dB on is the optimal strategy. The weak performance of *Greedy* can be explained from Fig. 6, where *Greedy* achieves high throughput until the first receiver finishes at less than approximately 18 time slots. The second receiver is still far from finishing as evidenced by the throughput curve flattening out around 30 time slots. *FH-OMB* performs close but is sub-optimal compared to *Dyn-Prog*, since the expected completion time is slightly inaccurate. The comparison in Fig. 7 shows that the expected completion time of *FH-OMB* algorithm (right) is less "round" than that of *Dyn-Prog* (left). Thus *FH-OMB* is more conservative and it sacrifices higher instantaneous rates to ensure that the relative difference in receiver state does not diverge too much.

To provide further insights into the behavior of the algorithms we show the state space visits in Fig. 8–11. As expected *Broadcast* keeps the two receivers very close to the diagonal where both receivers have the same amount of data, and slight deviations from the diagonal are only due to packet loss. *Greedy* in contrast makes quick progress until the second receiver finishes and for the remaining time only has the first receiver to serve. In fact, the steps with which the good receiver makes progress with *Greedy* can clearly be seen in Fig. 9. *FH-OMB* serves receivers similar to *Dyn-Prog* early on but then becomes too conservative as the good receiver progresses and beamforms more to the lagging receiver to balance receiver states.

### *B. Multipath Rayleigh Fading*

In this section, we show simulation results for a flat multipath Rayleigh fading channel, where the channel does not change within a time slot. The Doppler shift for the Rayleigh channel is set to 10Hz, corresponding to a slow fading channel for receivers moving at walking speed. Receivers are randomly distributed within the coverage area. The BS transmit power is set to 43dBm. With this, a cell edge receiver that is 250m from the BS is able to receive a packet with the lowest MCS with an average probability of $30\%$. The block size $B$ is set to 6400kbits.

We study the impact of increasing the number of receivers $N$ from 2 to 64 with different number of beamforming lobes (i.e., $K = \{2, 4, 8, 16\}$), again for a heterogeneous and a homogeneous scenario. Note that due to the high complexity of *Dyn-Prog*, we only compare the performance of the *FH-OMB* heuristic with that of *Broadcast* and *Greedy*.

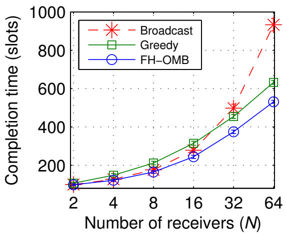


Fig. 12. Random receiver distribution, $K = 8$, $B = 6400$kbits.

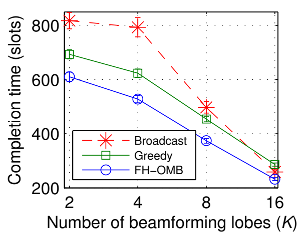


Fig. 13. Random receiver distribution, $N = 32$, $B = 6400$kbits.

*1) Random Receiver Distribution:* We first discuss a heterogeneous scenario, where $N = \{2, 4, 8, 16, 32, 64\}$ receivers are randomly distributed within the cell area of radius 250m and for $K = 8$ beamforming lobes. The performance depends significantly on the specific receiver distribution, in particular for smaller numbers of receivers. For up to 16 receivers, *Broadcast* performs almost as good as *FH-OMB* since there is a high probability that there is one receiver with a significantly worse channel than the others (see Fig. 12). As the number of receivers increases, a higher number of receivers see similar channel conditions and as in the previous two-channel scenario, the performance of *Broadcast* degrades since it does not exploit opportunistic gain. However, in this heterogeneous scenario this effect occurs mainly for $N > 32$ receivers, where *Broadcast*'s performance is significantly worse than that of *Greedy* and *FH-OMB*. *Greedy* performs worse than *Broadcast* for small $N$ for the same reason as above. The scenario is so small that the receivers are all very heterogeneous. As homogeneity increases for higher network densities, exploiting opportunistic gain becomes more important and *Greedy* outperforms *Broadcast*. *FH-OMB* performs well for all sizes of the receiver set. Its state-based completion time estimation results in the right tradeoff between opportunistic gain and multicasting gain and provides the lowest completion times of all approaches. It consistently outperforms *Greedy* by $9\%$ to $29\%$. The performance gain over *Broadcast* ranges from $1\%$ to $76\%$.

Next, we look at the impact of varying $K$ for a fixed $N = 32$. When increasing the number of beamforming lobes $K$, the array gain of the single lobe beam increases as well. In the specific antenna configuration that is chosen for our simulation, the array gains for $K = \{2, 4, 8, 16\}$ are 1.9, 3.4, 6.6 and 11.4, respectively. (Note that the array gain is not linear in $K$.) Therefore, as observed from Fig. 13, completion time decreases with increasing $K$ with respect to the achievable gain. *FH-OMB* outperforms both *Greedy* and *Broadcast* for all $K$. However, increasing $K$ has a more significant impact on the completion time of *Broadcast* than on *Greedy* and *FH-OMB*. For low $K$ and a wider beamwidth, *Broadcast* is limited by the receiver with the lowest SNR in each beam. (Also, a significant amount of the radiated energy does not cover any receiver.) As $K$ increases, fewer and fewer receivers are covered by a beam and in the extreme case of a single beam per receiver, *Broadcast* manages to perfectly balance the SNRs at the receivers (i.e, no energy is wasted by having a higher than necessary SNR at any receiver). Hence, *Broadcast*'s performance becomes closer and closer to *FH-OMB*. In contrast, *Greedy* may still beamform to a few receivers with high SNRs so that those finish first, before serving receivers with lower SNRs. In short, in heterogeneous scenarios with sufficient $K$, *Broadcast* that favors the weaker receivers by multicasting to all the receivers performs better than *Greedy* that capitalizes in maximizing opportunistic gain.

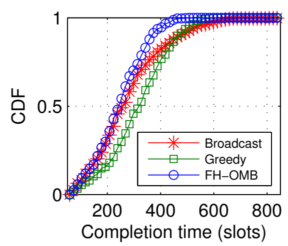


Fig. 14. CDF of completion time for random receiver distribution. $N = 16$, $K = 8$, $B = 6400$kbits.

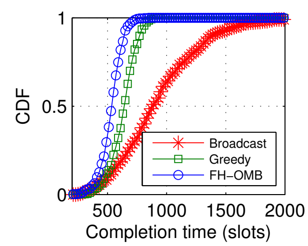


Fig. 15. CDF of completion time for random receiver distribution. $N = 64$, $K = 8$, $B = 6400$kbits.

To shed more light on the behavior of the algorithm, we show the CDF of completion time for the simulation runs for $K = 8$ and with $N = 16$ and $N = 32$ receivers in Fig. 14 and Fig. 15, respectively. In Fig. 14, *Broadcast* and *Greedy* have relatively similar completion time as *FH-OMB* in $10\%$ of the simulation runs. This happens in scenarios where all receivers are distributed quite close to the BS and thus all receivers have a relatively homogeneous good average channel quality. When receivers are distributed sparsely within the cell radius, with high probability they have different average channel qualities. Under this scenario, *Greedy* performs badly since it opportunistically serves the better receivers first and therefore results in higher completion times than both *FH-OMB* and *Broadcast*. Here, *FH-OMB* receivers finish at 465 slots for the worst scenarios, whereas *Greedy* and *Broadcast* both require 570 and 840 slots, respectively. When the number of receivers increases, Fig. 15 shows that *Broadcast* no longer has most of its completion time close to *FH-OMB* in most of the simulation runs. In fact, around $20\%$ of *Broadcast*'s completion time is similar to *Greedy* due to the limited number of beamforming lobes ($K = 8$), which leads to low multi-lobe beam's SNR. *Broadcast* is particularly bad in scenarios where many of the receivers are relatively far from the BS (and thus more homogeneous). The worst case completion time of *FH-*

*OMB* is at 675 slots, while *Greedy* and *Broadcast* require 810 and 2400 slots, respectively.

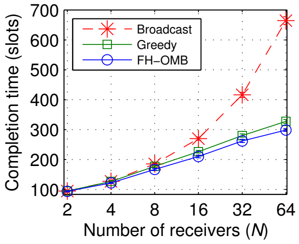


Fig. 16. Cell edge receiver distribution, $K = 8$, $B = 6400$kbits.

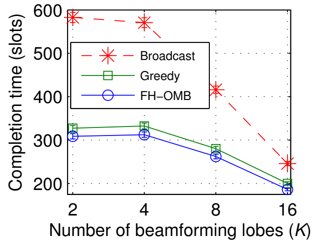


Fig. 17. Cell edge receiver distribution, $N = 32$, $B = 6400$kbits.

*2) Cell Edge Receiver Distribution:* In this scenario, receivers are all distributed close to the cell edge in the range of 190m to 220m and thus form a relatively homogeneous group. While such a scenario is less realistic than the one presented in Section V-B1, it is included to illustrate the performance degradation of the *Broadcast* algorithm in more homogeneous scenarios. Note that this performance is also indicative of the performance in heterogeneous scenarios with very high user densities, where many receivers are at the cell edge (see Fig. 12).

Here, the performance differences are much more drastic and *Broadcast* performs worse than the other schemes already for $N > 8$ (see Fig. 16). For 32 receivers, *FH-OMB* outperforms *Broadcast* by 59%. Although maximizing instantaneous throughput is the right strategy for homogeneous scenario, *FH-OMB* still manages to slightly outperform the *Greedy* algorithm by about $1 - 10\%$. Despite the homogeneity of the scenario, the slight differences among the receivers require a more sophisticated mechanism that does take states into account. Similar to the scenario with heterogeneous receiver distribution in Section V-B1, completion time improves with increasing $K$ (see Fig. 17) due to the higher effective SNR for each beam.

*3) Remarks:* Summarizing, we can observe that in the more realistic Rayleigh fading scenario, the performance gains of our *FH-OMB* heuristic are much more pronounced than in the simple scenarios with two receivers and two channel states. Compared to *Broadcast*, these gains increase as the number of receivers increases since *Broadcast* does not exploit opportunistic gain. On the other hand, *FH-OMB* is particularly beneficial over *Greedy* in heterogenous users scenarios because its balanced tradeoff between multi-user diversity and multicast gain results in lower completion times.

## VI. Conclusion

In this paper we studied opportunistic multicast beamforming for the finite horizon problem, where a base station has a fixed amount of erasure-coded data to transmit to multiple receivers. We model the problem as a dynamic programming problem to obtain the optimal solution. Due the high complexity of this approach, we design a heuristic algorithm, *FH-OMB*, that captures the characteristics of the optimal solution and provides a performance that is very close to it. We evaluate *FH-OMB*'s performance both for a discrete channel model as well as multipath Rayleigh fading. It outperforms other schemes based on maximizing the minimum SNR and broadcasting to all receivers (*Broadcast*), as well as greedily maximizing sum rate (*Greedy*). It improves performance by up to 76% over the former and up to 29% over the latter for heterogeneous scenarios with Rayleigh fading. For homogeneous scenarios, these gains are up to 122% and 10%, respectively. Similar (though slightly lower) gains are obtained for the simpler scenario with a discrete channel model.

For future work, we plan to address the impact of delayed and imperfect channel knowledge as well as optimize the feedback overhead.

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