Detecting Algorithmically Generated Malicious Domain Names

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ABSTRACT

Recent Botnets such as Conficker, Kraken and Torpig have used DNS based “domain fluxing” for command-and-control, where each Bot queries for existence of a series of domain names and the owner has to register only one such domain name. In this paper, we develop a methodology to detect such “domain fluxes” in DNS traffic by looking for patterns inherent to domain names that are generated algorithmically, in contrast to those generated by humans. In particular, we look at distribution of alphanumeric characters as well as bigrams in all domains that are mapped to the same set of IP-addresses. We present and compare the performance of several distance metrics, including KL-distance, Edit distance and Jaccard measure. We train by using a good data set of domains obtained via a crawl of domains and queries each of them until one of them is resolved and that the bot contacts the corresponding IP-address obtained that is typically used to host the command-and-control (C&C) server. Besides for command-and-control, spammers also routinely generate random domain names in order to avoid detection. For instance, spammers advertise randomly generated domain names in their spam emails to avoid detection by regular expression based domain blacklists that maintain signatures for recently ‘spamvertised’ domain names.

The botnets that have used random domain name generation vary widely in the random word generation algorithm as well as the way it is seeded. For instance, Conficker-A [27] bots generate 250 domains every three hours while using the current date and time at UTC (in seconds) as the seed, which in turn is obtained by sending empty HTTP GET queries to a few legitimate sites such as google.com, baidu.com, answers.com etc. This way, all bots would generate the same domain names every day. In order to make it harder for a security vendor to pre-register the domain names, the next version, Conficker-C [28] increased the number of randomly generated domain names per bot to 50K. Torpig [30, 6] bots employ an interesting trick where the seed for the random string generator is based on one of the most popular trending topics in Twitter. Kraken employs a much more sophisticated random word generator and constructs English-language alike words with properly matched vowels and consonants. Moreover, the randomly generated word is combined with a suffix chosen randomly from a pool of common English nouns, verbs, adjective and adverb suffixes, such as -able, -dom, -hood, -ment, -ship, or -ly.

From the point of view of botnet owner, the economics work out quite well. They only have to register one or a few domains out of the several domains that each bot would query every day. Whereas, security vendors would have to pre-register all the domains that a bot queries every day, even before the botnet owner registers them. In all the cases above, the security vendors had to reverse engineer the bot executable to derive the exact algorithm being used for generating domain names. In some cases, their algorithm would predict domains successfully until the botnet owner would

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patch all his bots with a repurposed executable with a different domain generation algorithm [30].

We argue that reverse engineering of botnet executables is resource- and time-intensive and precious time may be lost before the domain generation algorithm is cracked and consequently before such domain names are being generated algorithmically as a line of first defense. In this regards, we raise the following question: can we detect algorithmically generated domain names while monitoring DNS traffic even when a reverse engineered domain generation algorithm may not be available?

Hence, we propose a methodology that analyzes DNS traffic to detect if and when domain names are being generated algorithmically as a line of first defense. In this regards, our proposed methodology can point to the presence of bots within a network and the network administrator can disconnect bots from their C&C server by filtering out DNS queries to such algorithmically generated domain names.

Our proposed methodology is based on the following observation: current botnets do not use well formed and pronounceable language words since the likelihood that such a word is already registered at a domain registrar is very high; which could be self-defeating as the botnet owner would then not be able to control his bots. In turn this means that such algorithmically generated domain names can be expected to exhibit characteristics vastly different from legitimate domain names. Hence, we develop metrics using techniques from signal detection theory and statistical learning which can detect algorithmically generated domain names that may be generated via a myriad of techniques: (i) those generated via pseudo-random string generation algorithms as well as (ii) dictionary-based generators, for instance the one used by Kraken [5, 3, 4] as well as a publicly available tool, Kwyjibbo [12] which can generate words that are pronounceable yet not in the english dictionary.

Our method of detection comprises of two parts. First, we propose several ways to group together DNS queries: (i) either by the Top Level Domain (TLD) they all correspond to or; (ii) the IP-address that they are mapped to or; (iii) the connected component that they belong to, as determined via connected component analysis of the IP-domain bipartite graph. Second, for each such group, we compute metrics that characterize the distribution of the alphanumeric characters or bigrams (two consecutive alphanumeric characters) within the set of domain names. Specifically, we propose the following metrics to quickly differentiate a set of legitimate domain names from malicious ones: (i) Information entropy of the distribution of alphanumeric unigrams and bigrams) within a group of domains; (ii) Jaccard index to compare the set of bigrams between a malicious domain name with good domains and; (iii) Edit-distance which measures the number of character changes needed to convert one domain name to another.

We apply our methodology to a variety of data sets. First, we obtain a set of legitimate domain names via reverse DNS crawl of the entire IPv4 address space. Next, we obtain a set of malicious domain names as generated by Conficker, Kraken and Torpig as well as model a much more sophisticated domain name generation algorithm: Kwyjibbo [12]. Finally, we apply our methodology to one day of network traffic from one of the largest Tier-1 ISPs in Asia and South America and show how we can detect Conficker as well as a botnet hitherto unknown, which we call Mjugh (details in Section 5).

Our extensive experiments allow us to characterize the effectiveness of each metric in detecting algorithmically generated domain names in different attack scenarios. We model different attack intensities as number of domain names that an algorithm generates. For instance, in the extreme scenario that a botnet generates 50 domains mapped to the same TLD, we show that KL-divergence over unigrams achieves 100% detection accuracy albeit at 15% false positive rate (legitimate domain groups classified as algorithmic). We show how our detection improves significantly with much lower false positives as the number of words generated per TLD increases, e.g., when 200 domains are generated per TLD, then Edit distance achieves 100% detection accuracy with 8% false positives and when 500 domains are generated per TLD, Jaccard Index achieves 100% detection with 0% false positives.

Finally, our methodology of grouping together domains via connected components allows us to detect not only “domain fluxing” but also if it was used in combination with “IP fluxing”. Moreover, computing the metrics over components yields better and faster detection than other grouping methods. Intuitively, even if botnets were to generate random words and combine them with multiple TLDs in order to spread the domain names thus generated (potentially to evade detection), as long as they map these domains such that at least one IP-address is shared in common, then they reveal a group structure that can be exploited by our methodology for quick detection. We show that per-component analysis detects 26.32% more IP addresses than using per-IP analysis and 16.13% more hostnames than using per-domain analysis when we applied our methodology to detect Conficker in a Tier-1 ISP trace.

The rest of this paper is organized as follows. In Section 2, we compare our work against related literature. In Section 3, we present our detection methodology and introduce the metrics we have developed. In Section 4, we present the various ways by which domains can be grouped in order to compute the different metrics over them. Next, in Section 5, we present results to compare each metric as applied to different data sets and trace data. Further, in Section 6, we present the detection of malicious domains in a supervised learning framework, in particular, L1-regularized linear regression. We present a discussion over the relative computational complexity of each metric and the usefulness of component analysis in Section 7. Finally, in Section 8 we conclude.

2. RELATED WORK

Characteristics, such as IP addresses, whois records and lexical features of phishing and non-phishing URLs have been analyzed by McGrath and Gupta [22]. They observed that the different URLs exhibited different alphabet distributions. Our work builds on this earlier work and develops techniques for identifying domains employing algorithmically generated names, potentially for “domain fluxing”. Ma, et al [17], employ statistical learning techniques based on lexical features (length of domain names, host names, number of dots in the URL etc.) and other features of URLs to automatically determine if a URL is malicious, i.e., used for phishing or advertising spam. While they classify each URL independently, our work is focused on classifying a group of URLs as algorithmically generated or not, solely by making use of the set of alphanumeric characters used.
In addition, we experimentally compare against their lexical features in Section 5 and show that our alphanumeric distribution based features can detect algorithmically generated domain names with lower false positives than lexical features. Overall, we consider our work as complimentary and synergistic to the approach in [17].

With reference to the practice of “IP fast fluxing”, e.g., where the botnet owner constantly keeps changing the IP-addresses mapped to a C&C server, [24] implements a detection mechanism based on passive DNS traffic analysis. In our work, we present a methodology to detect cases where botnet owners may use a combination of both domain fluxing with IP fluxing, by having bots query a series of domain names and at the same time map a few of those domain names to an evolving set of IP-addresses. Also earlier papers [23, 20] have analyzed the inner-working of IP fast flux networks for hiding spam and scam infrastructure. With regards to botnet detection, [14, 15] perform correlation of network activity in time and space at campus network edges, and Xie et al in [33] focus on detecting spamming botnets by developing regular expression based signatures from a dataset of spam URLs.

We find that graph analysis of IP addresses and domain names embedded in DNS queries and replies reveal interesting macro relationships between different entities and enable identification of bot networks (Conficker) that seemed to span many domains and TLDs. With reference to graph based analysis, [34] utilizes rapid changes in user-bot graphs structure to detect botnet accounts.

Statistical and learning techniques have been employed by various studies for prediction [10, 25, 13]. We employed results from detection theory in designing our strategies for classification [31, 11].

Several studies have looked at understanding and reverse-engineering the inner workings of botnets [5, 3, 4, 16, 30, 26, 29]. Botlab has carried out an extensive analysis of several bot networks through active participation [19] and provided us with many example datasets for malicious domains.

### 3. DETECTION METRICS

In this section, we present our detection methodology that is based on computing the distribution of alphanumeric characters for groups of domains. First, we motivate our metrics by showing how algorithmically generated domain names differ from legitimate ones in terms of distribution of alphanumeric characters. Next, we present our three metrics, namely Kullback-Leibler (KL) distance, Jaccard Index (JI) measure and Edit distance. Finally, in Section 4 we present the methodology to group domain names.

#### 3.1 Data Sets

We first describe the data sets and how we obtained them:

(i) **Non-malicious ISP Dataset**: We use network traffic trace collected from across 100+ router links at a Tier-1 ISP in Asia. The trace is one day long and provides details of DNS requests and corresponding replies. There are about 270,000 DNS name server replies.

(ii) **Non-malicious DNS Dataset**: We performed a reverse DNS crawl of the entire IPv4 address space to obtain a list of domain names and their corresponding IP-addresses. We further divided this data set in to several parts, each comprising of domains which had 500, 200, 100 and 50 domain labels. The DNS Dataset is considered as non-malicious for the following reasons. Botnets may own only a limited number of IP addresses. Based on our study, we find that a DNS PTR request maps an IP address to only one domain name. The dataset thus obtained will contain very few malicious domain names per analyzed group. In the event that the bots exhibit IP fluxing, it is noteworthy that the botnet owners cannot change the PTR DNS mapping for IP addresses not owned. Although, the malicious name servers may point to any IP address.

(iii) **Malicious datasets**: We obtained the list of domain names that were known to have been generated by recent Botnets: Conficker [27, 28], Torpig [30] and Kraken [5, 3]. As described earlier in the Introduction, Kraken exhibits the most sophisticated domain generator by carefully matching the frequency of occurrence of vowels and consonants as well as concatenating the resulting word with common suffixes in the end such as -able, -dom, etc. (iv) **Kwyjibo**: We model a much more sophisticated algorithmic domain name generation algorithm by using a publicly available tool, Kwyjibo [12] which generates domain names that are pronounceable yet not in the English language dictionary and hence much more likely to be available for registration at a domain registrar. The algorithm uses a syllable generator, where they first learn the frequency of one syllable following another in words in English dictionary and then automatically generate pronounceable words by modeling it as a Markov process.

#### 3.2 Motivation

Our detection methodology is based on the observation that algorithmically generated domains differ significantly from legitimate (human) generated ones in terms of the distribution of alphanumeric characters. Figure 1(a) shows the distribution of alphanumeric characters, defined as the set of English alphabets (a-z) and digits (0-9) for both legitimate as well as malicious domains.

We derive the following points:

(i) **First**, note that both the non-malicious data sets exhibit a non-uniform frequency distribution, e.g., letters ‘m’ and ‘o’ appear most frequently in the non-malicious ISP data set whereas the letter ‘s’ appears most frequently in the non-malicious DNS data set. **(ii)** Even the most sophisticated algorithmic domain generator seen in the wild for Kraken botnet has a fairly uniform distribution, albeit with higher frequencies at the vowels: ‘a’, ‘e’ and ‘i’. **(iii)** If botnets of future were to evolve and construct words that are pronounceable yet not in the dictionary, then they would not exhibit a uniform distribution as expected. For instance, Kwyjibo exhibits higher frequencies at alphabets, ‘e’, ‘g’, ‘i’, ‘l’, ‘n’, etc. In this regards, techniques that are based on only the distribution of unigrams (single alphanumeric characters) may not be sufficient, as we will show through the rest of this section.

The terminology used in this and the following sections is as follows. For a hostname such as physics.university.edu, we refer to university as the second-level domain label, ed as the first-level domain, and university.edu as the second-level domain. Similarly, physics.university.edu is referred to as the third-level domain and physics is the third-level domain label. The ccTLDs such as co.uk are effectively considered as first-level domains.

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1 Even though domain names may contain characters such as -,-, we currently limit our study to alphanumeric characters only.
3.3 Metrics for anomaly detection

The K-L(Kullback-Leibler) divergence metric is a non-symmetric measure of "distance" between two probability distributions. The divergence (or distance) between two discretized distributions P and Q is given by: $D_{KL}(P||Q) = \sum_{i=1}^{n} P(i) \log \frac{P(i)}{Q(i)}$.

where $n$ is the number of possible values for a discrete random variable. The probability distribution $P$ represents the test distribution and the distribution $Q$ represents the base distribution from which the metric is computed.

Since the K-L measure is asymmetric, we use a symmetric form of the metric, which helps us deal with the possibility of singular probabilities in either distribution. The modified K-L metric is computed using the formula: $D_{sym}(PQ) = \frac{1}{2}(D_{KL}(P||Q) + D_{KL}(Q||P))$.

Given a test distribution $g$ computed for the domain to be tested, and non-malicious and malicious probability distribution over the alphanumerics as $g$ and $b$ respectively, we characterize the distribution as malicious or not via the following optimal classifier (for proof see appendix):

$$D_{sym}(gb) - D_{sym}(gg) \geq 0$$  \hspace{1cm} (1)

For the test distribution $g$ to be classified as non-malicious, we expect $D_{sym}(gg)$ to be less than $D_{sym}(gb)$. However, if $D_{sym}(gg)$ is greater than $D_{sym}(gb)$, the distribution is classified as malicious.

3.3.1 Measuring K-L divergence with unigrams

The first metric we design measures the KL-divergence of unigrams by considering all domain names that belong to the same group, e.g. all domains that map to the same IP-address or those that belong to the same top-level domain. We postpone discussion of groups to Section 4. Given a group of domains for which we want to establish whether they were generated algorithmically or not, we first compute the distribution of alphanumerics characters to obtain the distribution. Next, we compute the KL-divergence with a good distribution obtained from the non-malicious data sets (ISP or DNS crawl) and a malicious distribution obtained by modeling a botnet that uses alphanumerics uniformly. As expected, a simple unigram based technique may not suffice, especially to detect Kraken or Kwyjibo generated domains. Hence, we consider bigrams in our next metric.

3.3.2 Measuring K-L divergence with bigrams

A simple obfuscation technique that can be employed by algorithmically generated malicious domain names could be to generate domain names by using the same distribution of alphanumerics as commonly seen for legitimate domains. Hence, in our next metric, we consider distribution of bigrams, i.e., two consecutive characters. We argue that it would be harder for an algorithm to generate domain names that exactly preserve a bigram distribution similar to legitimate domains since the algorithm would need to consider the previous character already generated while generating the current character. The choices for the current character will be hence more restrictive than when choosing characters based on unigram distributions. Thus, the probability of test bigrams matching a non-malicious bigram distribution, becomes smaller.

Analogous to the case above, given a group of domains, we extract the set of bigrams present in it to form a bigram distribution. Note that for the set of alphanumerics characters that we consider [a-z, 0-9], the total number of bigrams possible are 36x36, i.e., 1,296. Our improved hypothesis now involves validating a given test bigram distribution against the bigram distribution of non-malicious and malicious domain labels. We use the database of non-malicious words to determine a non-malicious probability distribution. For a sample malicious distribution, we generate bigrams randomly. Here as well, we use KL-divergence over the bigram distribution to determine if a test distribution is malicious or legitimate.

3.3.3 Using Jaccard Index between bigrams

We present the second metric to measure the similarity
between a known set of components and a test distribution, namely the Jaccard index measure. The metric is defined as

\[ JI = \frac{A \cap B}{A \cup B} \]

where, \( A \) and \( B \) each represent the set of random variables. For our particular case, the set comprises of bigrams that compose a domain label or a hostname. Note that Jaccard index (JI) measure based on bigrams is a commonly used technique for web search engine spell-checking [21].

The core motivation behind using the JI measure is same as that for KL-divergence. We expect that bigrams occurring in randomized (or malicious) hostnames to be mostly different when compared with the set of non-malicious bigrams. To elaborate, we construct a database of bigrams which point to lists of non-malicious words, domain labels or hostnames, as the case may be. Now for each sub-domain present in a test set, we determine all non-malicious words that contain at least 75% of the bigrams present in the test word. Such a threshold helps us discard words with less similarity. However, longer test words may implicitly satisfy this criteria and may yield ambiguous JI value. As observed in section 5, the word sizes for 95% of non-malicious words do not exceed 24 characters, and hence we divide all test words into units of 24 character strings. Figure 2 presents the CDF of domain label sizes as observed in our DNS PTR dataset (described in section 5).

Calculating the JI measure is best explained with an example. Considering a randomized hostname such as `ickoxjsov.botnet.com`, we determine the JI value of the domain label `ickoxjsov` by first computing all bigrams (eight, in this case). Next, we examine each bigram’s queue of non-malicious domain labels, and short list words with at least 75% of bigrams, i.e., six of the eight bigrams. Words satisfying this criteria may include `thequickbrownfoxjumpsoverthelazydog` (35 bigrams). However, such a word still has a low JI value owing to the large number of bigrams in it. Therefore, the JI value is thus computed as \( \frac{6}{8 + 35 - 6} = 0.16 \). The low value indicates that the randomized test word does not match too well with the word from the non-malicious bigram database.

The JI measure is thus computed for the remaining words. The test words might comprise of a large number of bigrams and therefore do not always ensure a high JI value. We compute the JI measure using the equation described above and average it for all test words belonging to a particular group being analyzed. The averaged JI value for a non-malicious domain is expected to be higher than those for malicious groups.

As observed via our experiments in section 5, the JI measure is better at determining domain based anomalies. However, it is also computationally expensive as the database of non-malicious bigrams needs to be maintained in the memory. Also, classifying a non-malicious host will take more CPU cycles as we would obtain and compare a large set of words consisting of test word’s bigrams. Section 7 examines the computational complexity of various metrics that we use.

### 3.3.4 Edit distance

Note that the two metrics described earlier, rely on definition of a “good” distribution (KL-divergence) or database (JI measure). Hence, we define a third metric, Edit distance, which classifies a group of domains as malicious or legitimate by only looking at the domains within the group, and is hence not reliant on definition of a good database or distribution. The Edit distance between two strings represents an integral value identifying the number of transformations required to transform one string to another. It is a symmetric measure and provides a measure of intra-domain entropy. The type of eligible transformations are addition, deletion, and modification. For instance, to convert the word `cat` to `dog`, the edit distance is three as it requires all three characters to be replaced. With reference to determining anomalous domains, we expect that all domain labels (or hostnames) which are randomized, will, on an average, have higher edit distance value. We use the Levenshtein edit distance dynamic algorithm for determining anomalies. The algorithm for computing the Levenshtein edit distance has been shown in Algorithm 1 [21].

### Algorithm 1 Dynamic programming algorithm for finding the edit distance

\[
\text{EditDist}(s_1, s_2) =
\begin{cases}
0 & \text{if } i = 0 \text{ or } j = 0 \\
\min\begin{cases}
\text{EditDist}(s_1, s_2) & \text{if } i = j \\
\text{EditDist}(s_1, s_2) - 1 & \text{if } i > j \\
\text{EditDist}(s_1, s_2) - 1 & \text{if } i < j
\end{cases}
\end{cases}
\]

4. **GROUPING DOMAIN NAMES**

In this section, we present ways by which we group to-
geth other domain names in order to compute metrics that were defined in Section 3 earlier.

4.1 Per-domain analysis

Note that several botnets use several second-level domain names to generate algorithmic sub-domains. Hence, one way by which we group together domain names is via the second-level domain name. The intention is that if we begin seeing several algorithmically generated domain names being queried such that all of them correspond to the same second-level domain, then this may be reflective of a few favorite domain names to generate algorithmic sub-domains. Hence, one way by which we group together domain names is via the second-level domain name examplesite.org, def.example.org, etc., that have the same second-level domain name examplesite.org, we compute all the metrics over the alphanumeric characters and bigrams of the corresponding domain labels. Since domain fluxing involves a botnet generating a large number of domain names, we consider only domains which contain a sufficient number of third-level domain labels, e.g., 50, 100, 200 and 500 sub-domains.

4.2 Per-IP analysis

As a second method of grouping, we consider all domains that are mapped to the same IP-address. This would be reflective of a scenario where a botnet has registered several of the algorithmic domain names to the same IP-address of a command-and-control server. Determining if an IP address is mapped to several such malicious domains is useful as such an IP-address or its corresponding prefix can be quickly blacklisted in order to sever the traffic between a command-and-control server and its bots. We use the dataset from a Tier-1 ISP to determine all IP-addresses which have multiple hostnames mapped to it. For a large number of hostnames representing one IP address, we explore the above described metrics, and thus identify whether the IP address is malicious or not.

4.3 Component analysis

A few botnets have taken the idea of domain fluxing further and generate names that span multiple TLDs, e.g., Conficker-C generates domain names in 110 TLDs. At the same time domain fluxing can be combined with another technique, namely “IP fluxing” [24] where each domain name is mapped to an ever changing set of IP-addresses in an attempt to evade IP blacklists. Indeed, a combination of the two is even harder to detect. Hence, we propose the third method for grouping domain names into connected components.

We first construct a bipartite graph \( G \) with IP-addresses on one side and domain names on the other. An edge is constructed between a domain name and an IP-address if that IP-address was ever returned as one of the responses in a DNS query. When multiple IP addresses are returned, we draw edges between all the returned IP addresses and the queried host name.

First, we determine the connected components of the bipartite graph \( G \), where a connected component is defined as one which does not have any edges with any other components. Next, we compute the various metrics (KL-divergence for unigrams and bigrams, JI measure for bigrams, Edit distance) for each component by considering all the domain names within a component.

Component extraction separates the IP-domain graph into components which can be classified into the following classes:

(i) IP fan: these have one IP-address which is mapped to several domain names. Besides the case where one IP-address is mapped to several algorithmic domains, there are several legitimate scenarios possible. First, this class could include domain hosting services where one IP-address is used to provide hosting to several domains, e.g., Google Sites, etc. Other examples could be mail relay service where one mail server is used to provide mail relay for several MX domains. Another example could be when domain registrars provide domain parking services, i.e., someone can purchase a domain name while asking the registrar to host it temporarily.

(ii) Domain fan: these consist of one domain name connected to multiple IPs. This class will contain components belonging to the legitimate content providers such as Google, Yahoo, etc.

(iii) Many-to-many component: these are components that have multiple IP addresses and multiple domain names, e.g., Content Distribution Networks (CDNs) such as Akamai.

In section 6, we briefly explain the classification algorithm that we use to classify test components as malicious or not.

5. RESULTS

In this section, we present results of employing various metrics across different groups, as described in section 3 and 4. We briefly describe the data set used for each experiment. With all our experiments, we present the results based on the consideration of increasing number of domain labels. In general, we observe that using a larger test data set yields better results.

5.1 Per-domain analysis

5.1.1 Data set

The analysis in this sub-section is based only on the domain labels belonging to a domain. The non-malicious distribution \( g \) may be obtained from various sources. For our analysis, we use a database of DNS PTR records corresponding to all IPv4 addresses. The database contains 659 second-level domains with at least 50 third-level sub-domains, while there are 103 second-level domains with at least 500 third-level sub-domains. From the database, we extract all second-level domains which have at least 50 third-level sub-domains. All third-level domain labels corresponding to such domains are used to generate the distribution \( g \). For instance, a second-level domain such as university.edu may have many third-level domain labels such as physics, cse, humanities etc. We use all such labels that belong to trusted domains, for determining \( g \).

To generate a malicious base distribution \( b \), we randomly generate as many characters as present in the non-malicious distribution. We use domain labels belonging to well-known malware based domains identified by Botlab, and also a publicly available spambase database, as malicious domains [1, 9] for verification using our metrics. Botlab provides us with various domains used by Kraken, Pushdo, Storm, MegaD, and Srizbi [1]. For per-domain analysis, the test words used are the third-level domain labels.

Figure 1(a) shows how malicious/non-malicious distributions appear for the DNS PTR dataset as well as the ISP dataset described in the following sections.

We will present the results for all the four measures de-
scribed earlier, for domain-based analysis. In later sections, we will only present data from one of the measures for brevity.

5.1.2 K-L divergence with unigram distribution

We measure the symmetrized K-L distance metric from the test domain to the malicious/non-malicious alphabet distributions. We classify the test domain as malicious or non-malicious based on equation (10) in Appendix A. Figure 3(a) shows the results from our experiment presented as an ROC curve.

The figure shows that the different sizes of test data sets produce relatively different results. The area under the ROC is a measure of the goodness of the metric. We observe that with 200 or 500 domain labels, we cover a relatively greater area, implying that using many domain labels helps obtain accurate results. For example, using 500 labels, we obtain 100% detection rate with only 2.5% false positive rate. Note that with a larger data set, we indeed expect higher true positive rates for small false positive rates, as larger samples will stabilize the evaluated metrics.

The number of domain labels required for accurate detection corresponds to the latency of accurately classifying a previously unseen domain. The results suggest that a domain-fluxing domain can be accurately characterized by the time it generates around 500 names.

5.1.3 K-L divergence with bigram distribution

Figure 3(b) presents the results of employing K-L distance metric over bigram distributions. We observe again that using 200 or 500 domain labels does better than using smaller number of labels, with 500 labels doing the best. Experiments with 50/100 domain labels yield similar results.

We note that the performance with unigram distributions is slightly better than using bigram distributions. However, when botnets employ counter measures to our techniques, the bigram distributions may provide better defense compared to unigram distributions as they require more effort to match the good distribution (g).

5.1.4 Jaccard measure of bigrams

The Jaccard Index measure does significantly better in
comparison to the previous metrics. From figure 4(a), it is evident that using 500 domain labels gives us a clear separation for classification of test domains (and hence an area of 1). Using 50 or 100 labels is fairly equivalent with 200 labels doing comparatively better. The JI measure produces higher false positives for smaller number of domains (50/100/200) than K-L distance measures.

5.1.5 Edit distance of domain labels

Figure 4(b) shows the performance using edit distance as the evaluation metric. The detection rate for 50/100 test words reaches 1 only for high false positive rates, indicating that a larger test word set should be used. For 200/500 domain labels, 100% detection rate is achieved at false positive rates of 5-7%.

5.1.6 Kwyjibo domain label analysis

Kwyjibo is a tool to generate random words which can be used as domain labels [12]. The generated words are seemingly closer to pronounceable words of the English language, in addition to being random. Thus many such words can be created in a short time. We anticipate that such a tool can be used by attackers to generate domain labels or domain names quickly with the aim of defeating our scheme. Therefore, we analyze Kwyjibo based words, considering them as domain labels belonging to a particular domain.

The names generated by Kwyjibo tool could be accurately characterized by our measures given sufficient names. Example results are presented in Fig. 5 with K-L distances over unigram distributions. From figure 5, we observe that verification with unigram frequency can lead to a high detection rate with very low false positive rate. Again, the performance using 500 labels is the best. We also observe a very steep rise in detection rates for all the cases. The Kwyjibo domains could be accurately characterized with false positive rates of 6% or less.

The initial detection rate for Kwyjibo is low as compared to the per-domain analysis. This is because the presence of highly probable non-malicious unigrams in Kwyjibo based domains makes detection difficult at lower false positive rates. The results with other measures (K-L distance over bigram distributions, JI and edit distances) were similar: kwyjibo domains could be accurately characterized at false positive rates in the range of 10-12%, but detection rates were nearly zero at false positive rates of 10% or less.

The scatter plot presented in Fig. 6 indicates the clear separation obtained between non-malicious and malicious domains. The plot represents the Jaccard measure using 500 test words. We highlight the detection of botnet based malicious domains such as Kraken, MegaD, Pushdo, Srizbi, and Storm. A few well-known non-malicious domains such as apple.com, cisco.com, stanford.edu, mit.edu, and yahoo.com have also been indicated for comparison purposes.

5.1.7 Progressive demarcation

The earlier results have showed that very high good detection rates can be obtained at low false positive rates once we have 500 or more hostnames of a test domain. As discussed earlier, the number of hostnames required for our analysis corresponds to latency of accurately characterizing a previously unseen domain. During our experiments, not all the test domains required 500 hostnames for accurate characterization since the distributions were either very close to the good distribution or bad distribution. These test domains could be characterized with a smaller latency (or smaller number of hostnames).

In order to reduce the latency for such domains, we tried an experiment at progressive demarcation or characterization of the test domains. Intuitively, the idea is to draw two thresholds above one there are clearly good domains, below the second threshold there are clearly bad domains and the domains between the two thresholds require more data (or hostnames) for accurate characterization. These thresholds are progressively brought closer (or made tighter) as more hostnames become available, allowing more domains to be accurately characterized until we get 500 or more hostnames for each domain. The results of such an experiment using the JI measure are shown in Fig. 7.

We establish the lower bound using the formula \( \mu_g + \sigma_b \) where \( \mu_g \) is the mean of JI values observed for bad or malicious domains and \( \sigma_b \) is the standard deviation. Similarly, the upper bound is obtained using the expression \( \mu_g - \sigma_g \) where the subscript \( g \) implies good domains. Figure 7 shows the detection rate for the considered domains.

![Figure 5: ROC curve : K-L metric with unigram distribution (Kwyjibo).](image)

![Figure 6: Scatter plot with Jaccard Index for bi-grams (500 test words).](image)
We see a monotonically increasing detection rate for both good and bad domains. It is observed that 85% of bad domains could be so characterized accurately with only 100 hostnames while only about 23% of good domains can be so characterized with 100 hostnames. In addition, our experiments indicate that only a small percentage of domains require 200 or more hostnames for their characterization.

![Figure 7: Illustrating benefits of progressive demarcation with JI measure.](image)

5.2 Per-IP analysis

5.2.1 Data set

Here, we present the evaluation of domain names that map to an IP address. For analyzing the per-IP group, for all hostnames mapping to an IP-address, we use the domain labels except the top-level domain TLD as the test word. For instance, for hostnames `physics.university.edu` and `cse.university.edu` mapping to an IP address, say 6.6.6.6, we use `physicsuniversity` and `cseuniversity` as test words. However, we only consider IP addresses with at least 50 hostnames mapping to it. We found 341 such IP addresses, of which 53 were found to be malicious, and 288 were considered non-malicious. The data is obtained from DNS traces of a Tier-1 ISP in Asia.

Many hostnames may map to the same IP address. Such a mapping holds for botnets or other malicious entities utilizing a large set of hostnames mapping to fewer C&C(Command and Control) servers. It may also be valid for legitimate internet service such as for Content Delivery Networks (CDNs).

5.3 Summary

For a larger set of test words, the relative order of efficacy of different measures decreases from JI, to edit distance to...
K-L distances over bigrams and unigrams. However, interestingly, we observe the exact opposite order when using a small set of test words. For instance, with 50 test words used for the per-domain analysis, the false positive rates at which we obtain 100% detection rates, are approximately 50% (JI), 20% (ED), 25% (K-L with bigram distribution), and 15% (K-L with unigram distribution). Even though the proof in the Appendix indicates that K-L divergence is an optimal metric for classification, in practice, it does not hold as the proof is based on the assumption that it is equally likely to draw a test distribution from a good or a bad distribution.

6. DETECTION VIA SUPERVISED LEARNING

As discussed in Section 5.3 immediately above, the relative merits of each measure vary depending, for instance, on the number of subdomains present in a domain being tested. In this section, we formulate detection of malicious domains (algorithmically generated) as a supervised learning problem such that we can combine the benefits afforded by each measure while learning the relative weights of each measure during a training phase. We divide the one-day long trace from the South Asian Tier-1 ISP in to two halves such that the first one of 10 hours duration is used for training. We test the learnt model on the remainder of the trace from South Asian ISP as well as over a different trace from a Tier-1 ISP in South America. In this section, we use the grouping methodology of connected components, where all “domain name, response IP-address” pairs present during a time window (either during training or test phases) are grouped in to connected components.

6.1 L1-regularized Linear Regression

We formulate the problem of classifying a component as malicious (algorithmically generated) or legitimate in a supervised learning setting as a linear regression or classification problem. We first label all domains within the components found in the training data set by querying against domain reputation sites such as McAfee Site Advisor [2] and Web of Trust [7], as well as by searching for the URLs on search engines [32]. Next, we label a component as good or malicious, and we find 27 of them to be malicious after cross checking with external sources (Web of Trust, McAfee, etc.) while two components (99 domains) are false positives and comprise of Google and domains belonging to news blogs.

On applying our model to the rest of the trace, 29 components (out of a total of 3K components) are classified as malicious, and we find 27 of them to be malicious after cross checking with external sources (Web of Trust, McAfee, etc.) while two components (99 domains) are false positives and comprise of Google and domains belonging to news blogs.
Note that here we use a broad definition of malicious domains as those that could be used for any nefarious purposes on the web, i.e., we do not necessarily restrict the definition to only include botnet domain generation algorithm. Out of the 27 components that were classified as malicious, one of them corresponds to the Conficker botnet, which is as expected since our training incorporated features learnt from Conficker. We next provide details on the remaining 26 components that were determined as malicious (see Table 2).

Mjuyh Botnet: The most interesting discovery from our component analysis is that of another Botnet, which we call Mjuyh, since they use the domain name mjuyh.com (see Table 3). The fourth-level domain label is generated randomly and is 57 characters long. Each of the 121 domain names belonging to this bot network return 10 different IP addresses on a DNS query for a total of 1.2K IP-addresses. Also, in some replies, there are invalid IP addresses like 0.116.157.148. All the 10 IP addresses returned for a given domain name, belong to different network prefixes. Furthermore, there is no intersection in the network prefixes between the different domain names of the mjuyh bot. We strongly suspect that this is a case of “domain fluxing” along with “IP fast fluxing”, where each bot generated a different randomized query which was resolved to a different set of IP-addresses.

Helldark Trojan: We discovered a component containing five different third-level domains (a few sample domain names are as shown in Table 3) The component comprises of 28 different domain names which were all found to be spreading multiple Trojans. One such Trojan spread by these domains is Win32/Hamweq.CW that spreads via removable drives, such as USB memory sticks. They also have an IRC-based backdoor, which may be used by a remote attacker directing the affected machine to participate in Distributed Denial of Service attacks, or to download and execute arbitrary files [8].

Mis-spelt component: There are about five components (comprising 220 domain names) which used tricked (mis-spelt or slightly different spelling) names of reputed domain names. For example, these components use domain names such as uahoo.co.uk to trick users trying to visit yahoo.co.uk (since the alphabet ‘u’ is next to the alphabet ‘y’, they expect users to enter this domain name by mistake). Dizneyland.com is used to misdirect users trying to visit Disneyland.com (which replaces the alphabet ‘s’ with alphabet ‘s’). We still consider these components as malicious since they comprise of domains that exhibit unusual alphanumeric features.

Domain Parking: We found 15 components (630 domain names) that were being used for domain parking, i.e., a practice where users register for a domain name without actually using it, in which case the registrar’s IP-address is returned as the DNS response. In these 15 components, one belongs to GoDaddy (66 domain names), 13 of them belong to Sedo domain parking (510 domain names) and one component belongs to OpenDNS (57 domain names). Clearly these components represent something abnormal as there are many domains with widely disparate algorithmic features clustered together on account of the same IP-address they are mapped to.

Adult Content: We find four components that comprise of 349 domains primarily used for hosting adult content sites. Clearly this matches the well known fact, that in the world of adult site hosting, the same set of IP-addresses are used to host a vast number of domains, each of which in turn may use very different words in an attempt to drive traffic.

In addition, for comparison purposes, we used the lexical features of the domain names such as the length of the domain names, number of dots and the length of the second-level domain name (for example, yzc.com) for training on the same ISP trace, instead of using the KL-divergence, JI measure and Edit distance measures used in our study. These lexical features were found to be useful in an earlier study in identifying malicious URLs [17]. The model trained on these lexical features correctly labeled four components as malicious (Conficker bot network, three adult content components and one component containing mis-spelt domain names) during the testing phase, but it also resulted in 30 components which were legitimate as being labeled incorrectly; compare this against 27 components that were correctly classified as malicious and two that were false positives on using our alphanumeric features.

We also test our model on a trace obtained from a South America based Tier-1 ISP. This trace is about 20 hours long and is collected on a smaller scale as compared to the ISP trace from Asia. The time lag between the capture of S. American Tier-1 ISP trace and the previously used ISP trace from Asia, is about 15 days. We use the same training set for the prediction model as we use for the ISP trace from Asia. In the prediction stage, we successfully detect the Conficker component with no false positives. The Conficker component has 185 domain names and 10 IP addresses. Of the 10 IP addresses determined for the Conficker component of the South American trace, nine are common with the Asia ISP trace’s Conficker component. We conclude that Conficker based C&C servers have relatively large TTLs. However, out of the 185 domain names only five domains are common from this component and the component from the ISP trace from Asia. Clearly, the Conficker botnet exhibits rapid domain fluxing. Overall, this experiment shows that a training model learnt in one network can be applied to a completely different network and still successfully detect malicious domain groups.

7. DISCUSSION

7.1 Usefulness of component analysis

Conficker botnet, present in our ISP trace, employs domain fluxing across TLDs, that became directly visible after IP-domain components were extracted and analyzed from the trace. The component analysis allowed the application of our detection methods across several different domains, which otherwise would have been separated from each other. In addition, component analysis allowed us to detect Conficker domains that would not have been detectable with our approach when applied to domain names alone since some of these domains contained fewer than 50 names needed for accurate analysis. Similarly, some of the IP addresses in the component hosted fewer than 50 names and would not have been detected with the IP address based analysis either. However, these domains will be included in the component analysis as long as the component has altogether more than 50 names.

$L = D_1 D_2 \ldots D_n$ be the number of hostnames and $L$ be the number of IP addresses in the component. If $D_1, D_2$ are the number of hostnames and corresponding IP addresses detected
through domain level analysis, we define domain level completeness ratios as $D_d/D_s$ and $I_d/I_s$. Similarly, we can define the completeness ratios for IP-based analysis as $D_i/D_s$ and $I_i/I_s$, where $D_i$ and $I_i$ correspond to the total number of hostnames and IP addresses of the Conficker botnet detected by the IP-based analysis.

For the Conficker botnet, these completeness ratios for IP-based analysis were 73.68% for IP addresses and 98.56% for hostnames. This implies that we are able to detect an additional 26.32% of IP addresses and a relatively small fraction of 1.44% of hostnames for those IP addresses. The completeness ratios for domain based analysis were found to be 100% for IP addresses and 83.87% for the hostnames. Therefore, we do 16.13% better in terms of determining the hostnames using the per-domain analysis. This shows that the component level analysis provides additional value in analyzing the trace for malicious domains.

### 7.2 Complexity of various measures

Table 4 identifies the computational complexity for every metric, and for all groups that we use. We observe that K-L metrics analyzing unigram and bigram distributions can be computed fairly efficiently. However, for the JI measure, the size of the non-malicious database largely influences the time taken to compute the measure. A good database size results in a higher accuracy, at the cost of increased time taken for analysis. Similarly, edit distance takes longer for larger word lengths, and the number of test words. However, it is independent of any database, hence the space requirements are smaller.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Alphabet size</td>
</tr>
<tr>
<td>$W$</td>
<td>Maximum word size</td>
</tr>
<tr>
<td>$K$</td>
<td>Number of test words</td>
</tr>
<tr>
<td>$K'$</td>
<td>Number of test words in a component</td>
</tr>
<tr>
<td>$S_g$</td>
<td>Number of words in non-malicious database</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Grp.</th>
<th>K-L unigram</th>
<th>K-L bigram</th>
<th>JI</th>
<th>ED</th>
</tr>
</thead>
<tbody>
<tr>
<td>dom.</td>
<td>$O(KW + A)$</td>
<td>$O(KW + A^2)$</td>
<td>$O(KW^2 S_g)$</td>
<td>$O(K^2 W^2)$</td>
</tr>
<tr>
<td>IP</td>
<td>$O(KW + A)$</td>
<td>$O(KW + A^2)$</td>
<td>$O(KW^2 S_g)$</td>
<td>$O(K^2 W^2)$</td>
</tr>
<tr>
<td>Com.</td>
<td>$O(KW + A)$</td>
<td>$O(KW + A^2)$</td>
<td>$O(KW^2 S_g)$</td>
<td>$O(K^2 W^2)$</td>
</tr>
</tbody>
</table>

We briefly describe how we determine the bounds as expressed in table 4 for the per-domain group. For the K-L unigram analysis since we examine every character of every test word, the complexity is bounded by $KW$. We then compute, for every character in the alphabet $A$, the divergence values. Therefore, we obtain the complexity as $O(KW + A)$. Bigram distribution based K-L divergence is calculated similarly except that the new alphabet size is $A^2$. While calculating the Jaccard index, note that the number of bigrams obtained is $O(W - 1)$. For each bigram, we examine the queues pointing to words from the non-malicious database. Thus, for each bigram, we examine $O(W S_g)$ bigrams. Since we do it for $K$ test words, we obtain $O(KW^2 S_g)$. For every test word used while obtaining the edit distance, we examine it against the $K - 1$ test words. Therefore, the total complexity is simply $O(K W^2)$. The expressions for per-IP and per-component groups are obtained analogously.

It is interesting to note that $A$ is of the size 36 (0-9, a-z characters). $K$ used in our analysis varies as 50/100/200/500. However, the average value for $K$ is higher in comparison. The DNS PTR dataset considered for per-domain analysis has approximately 469,000 words used for training purposes. This helps us estimate $S_g$. For the ISP dataset, $S_g$ is of the order of 11522 words. An estimate of $W$ for the DNS PTR dataset is obtained from figure 2.

### 8. CONCLUSIONS

In this paper, we propose a methodology for detecting algorithmically generated domain names as used for “domain fluxing” by several recent Botnets. We propose statistical measures such as Kullback-Leibler divergence, Jaccard index, and Levenshtein edit distance for classifying a group of domains as malicious (algorithmically generated) or not. We perform a comprehensive analysis on several data sets including a set of legitimate domain names obtained via a crawl of IPv4 address space as well as DNS traffic from a Tier-1 ISP in Asia. One of our key contributions is the relative performance characterization of each metric in different scenarios. In general, the Jaccard measure performs the best, followed by the Edit distance measure, and finally the KL divergence. Furthermore, we show how our methodology when applied to the Tier-1 ISP’s trace was able to detect Conficker as well as a botnet yet unknown and unclassified, which we call as Mjugh. In this regards, our methodology can be used as a first alarm to indicate the presence of domain fluxing in a network, and thereafter a network security analyst can perform additional forensics to infer the exact algorithm being used to generate the domain names. As future work, we plan to generalize our metrics to work on n-grams for values of $n > 2$.

### 9. ACKNOWLEDGMENTS

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is classified as good, if it is more likely to have
such that $\mathbf{q}$ can belong to a

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**APPENDIX**

Let $\mathcal{A} = \{a_1, a_2, \ldots, a_M\}$ denote $M$ the letters of the alphabet from which the domain names are chosen (in our case, this is English alphabet with spaces and special characters). Let $\mathbf{g} = [g_1, g_2, \ldots, g_M]$ and $\mathbf{b} = [b_1, b_2, \ldots, b_M]$ be the distribution of the letters in the good and bad domains, respectively. Let $\mathbf{x}$ be the actual domain name of length $N$, that has to be classified as being good or bad. Let the letter $a_i$ appear $n_i$ times in $\mathbf{x}$ such that $\sum_i n_i = N$. Let $\mathbf{g} = [g_1, g_2, \ldots, g_M]$ be the distribution of the different letters in $\mathbf{x}$, i.e., $q_i = n_i/N$.

Under the assumption that *a priori*, $\mathbf{x}$ can belong to a good or bad domain with equal probability, the classifier that minimizes the probability of error (wrong classification) is given by the maximum-likelihood classifier which classifies $\mathbf{x}$ according to

$$P(\mathbf{g} | \mathbf{x}) \propto P(\mathbf{g} | \mathbf{b})$$

(3)

Intuitively, $\mathbf{x}$ is classified as good, if it is more likely to have resulted from the good distribution than from the bad distribution. The above classifier can be specified in terms of the likelihood ratio given by

$$\lambda(\mathbf{x}) = \frac{P(\mathbf{x} | \mathbf{g})}{P(\mathbf{x} | \mathbf{b})} \geq 1$$

(4)
As we will see later, it is easier to work with an equivalent quantity \( \frac{1}{N} \log \lambda (x) \). The classifier is then given according to

\[
\frac{1}{N} \log \lambda (x) = \frac{1}{N} \log \frac{P(x|g)}{P(x|b)} \geq 0 \tag{5}
\]

Under the assumption that the letters in \( x \) have been generated independently from the same distribution, \( P(x|g) \) is given by

\[
P(x|g) = \prod_{k=1}^{N} P(x_k|g) = \prod_{i=1}^{M} P(a_i|g)^{n_i} = \prod_{i=1}^{M} g_i^{n_i}.
\]

The second equality follows by grouping all the occurrences of the letters \( a_i \) together and recall that there are \( n_i \) such occurrences. Similarly,

\[
P(x|b) = \prod_{k=1}^{N} P(x_k|b) = \prod_{i=1}^{M} P(a_i|b)^{n_i} = \prod_{i=1}^{M} b_i^{n_i}.
\]

Using (6) and (7) in (5), the log-likelihood ratio can be seen to be

\[
\frac{1}{N} \log \lambda (x) = \frac{1}{N} \log \frac{P(x|g)}{P(x|b)} = \log \frac{\prod_{i=1}^{M} g_i^{n_i}}{\prod_{i=1}^{M} b_i^{n_i}} \tag{8}
\]

Dividing the numerator and the denominator by \( \prod_i q_i^{n_i} \), we get

\[
\frac{1}{N} \log \lambda (x) = \log \frac{\prod_{i=1}^{M} \left( \frac{g_i}{q_i} \right)^{n_i}}{\prod_{i=1}^{M} \left( \frac{b_i}{q_i} \right)^{n_i}} = \sum_i q_i \log \frac{g_i}{q_i} - \sum_i q_i \log \frac{b_i}{q_i} \tag{9}
\]

\[
= D(q|g) - D(q|b) \tag{10}
\]

where \( D(q|b) \) is the Kullback-Leibler (KL) distance between the two distributions. Thus, the optimal classifier given in (5) is equivalent to

\[
D(q|g) - D(q|b) \geq 0 \tag{11}
\]

This result is intuitively pleasing since the classifier essentially computes the KL “distance” between \( q \) and the two distributions and chooses the one that is ‘closer’.