Interval Scheduling to Maximize Bandwidth Provision

Mordechai Shalom*, Prudence W.H. Wong, Shmuel Zaks*****

** Tel Hai College, Upper Galilee, 12210, Israel, e-mail: cmshalom@telhai.ac.il*

*** Dept. of Computer Science, Univ. of Liverpool, Liverpool, UK, e-mail: pwong@liverpool.ac.uk*

**** Dept of Computer Science, Technion, Haifa, Israel, Catedra de Excelencia, Dept. de Ingenieria Telematica, Univ. Carlos III de Madrid, IMDEA Networks, Madrid, Spain, e-mail: zaks@cs.technion.ac.il*

We study an interval scheduling problem in which each job j is associated with a time interval I_j, a minimum a_j and maximum b_j required bandwidths, and a weight w_j. We are given W colors. We need to assign to each job j between a_j and b_j colors, such that each color is assigned at each time to at most one interval. The *weighted bandwidth* allocated to job j is the number of colors allocated to it times $|I_j|$, and the *weighted bandwidth of a coloring* is the sum of the weighted bandwidth of all jobs. The objective is to find a coloring with maximum weighted bandwidth. A coloring is *(circularly) contiguous* if the set of colors for each job forms an (circular) interval.

The problem is motivated by bandwidth allocation, which is common in network applications such as content distribution networks or mobile clients, which require bandwidth reservations to support hangovers for streaming video [1]. Our problem is motivated by the DWDM (dense wavelength division multiplexing) network [3, 4]. In optical networks, high-speed signals are sent through optical fibers using WDM technology. The spectrum of light that can be transmitted through the fiber is divided into frequency intervals. When the underlying network topology is a path, it is in analogy to the time line while the available wavelength is in analogy to the available colors in our scheduling problem.

For non-contiguous coloring we show a polynomial time algorithm that finds an optimal coloring when $a_j \geq 0$ and $b_j = W$ for all jobs j. For contiguous coloring we observe that our problem is NP-hard as it generalizes the dynamic storage allocation problem [2]. We show polynomial time dynamic programming optimal algorithm when the load is bounded. For circularly contiguous coloring we show a randomized polynomial time algorithm that converts a circularly contiguous coloring to a contiguous coloring whose weighted bandwidth is at least $\frac{3}{4}$ of the original one, when $a_j = 1$ for all j. Last we discuss a special instance termed *p*roper, with no job interval is properly contained in another one. We show a polynomial time optimal circularly contiguous coloring, and conclude with a polynomial time randomized contiguous coloring with approximation ratio $\frac{4}{3}$, when $a_j = 1$ for all j. These randomized algorithms can be derandomized.

References

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