Bisection Width of Multidimensional Product Graphs

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In this paper we will provide two general results that allow to obtain upper and lower bounds on the bisection width of a product graph as a function of some properties of its factor graphs. The most interesting contribution of this paper is the exact value of the bisection width of a d-dimensional torus, as this problem has been open for almost 20 years [2]. Our work is partially based on the work by Azizoğlu and Eğecioğlu. In [1] they study the relation between the isoperimetric number and the bisection width of different product networks and obtained and exact value for the bisection width of the d-dimensional array studying it as a product of paths. Similarly, we have been able to provide a lower and an upper bound for the bisection width of product graphs whose factor graphs have the same maximal congestion with multiplicity r, for the former case, or the same central cut, for the latter one.

The general results provided are used to obtain exact bounds on the bisection width of several product graphs. The factor graphs used are paths, rings, complete binary trees (CBTs), and extended trees (which are CBTs with the leaves connected as a path). Then, we show that the Cartesian product of rings (i.e., the torus) of sizes $k_1 \geq \ldots \geq k_d$ has bisection width $2\sum_{i=1}^{e} C_i$, where $C_i = \prod_{j=i+1}^{d} k_j$ for $i \in [1, e]$, and e being the lowest dimension with an even number of vertices. (If there is no such dimension, $e = d$.). Additionally, we show that the Cartesian product of a mixture of XTs and rings has the same bisection width. (When all factor graphs are XTs, $e = d$.) Finally, we show that the Cartesian product of a mixture of CBTs and paths has bisection width $\sum_{i=1}^{e} C_i$. (When all factor graphs are CBTs, $e = d$.)

References

[1] M. Cemil Azizoğlu and Ömer Eğecioğlu. The bisection width and the isoperimetric number of arrays. *Discrete Applied Mathematics*, 138(1-2):3–12, 2004.

[2] F. T. Leighton. *Introduction to parallel algorithms and architectures: array, trees, hypercubes*. Morgan Kaufmann Publishers Inc., San Francisco, CA, USA, 1992.