Performance of Overlaid MIMO Cellular Networks with TAS/MRC Under Hybrid-Access Small Cells and Poisson Field Interference

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***Abstract*—This paper presents new approaches to characterize the achieved performance of hybrid control-access small cells in the context of two-tier multi-input multi-output (MIMO) cellular networks with random interference distributions. The hybrid scheme at small cells (such as femtocells) allows for sharing radio resources between the two network tiers according to the densities of small cells and their associated users, as well as the observed interference power levels in the two network tiers. The analysis considers MIMO transceivers at all nodes, for which antenna arrays can be utilized to implement transmit antenna selection (TAS) and receive maximal ratio combining (MRC) under MIMO point-to-point channels. Moreover, it targets network-level models of interference sources inside each tier and between the two tiers, which are assumed to follow Poisson field processes. To fully capture the occasions for Poisson field distribution on MIMO spatial domain. Two practical scenarios of interference sources are addressed including highly-correlated or uncorrelated transmit antenna arrays of the serving macrocell base station. The analysis presents new analytical approaches that can characterize the downlink outage probability performance in any tier. Furthermore, the outage performance in high signal-to-noise (SNR) regime is also obtained, which can be useful to deduce diversity and/or coding gains.**

I. Introduction

The high demand on wireless services has motivated the search for improved data driven cellular standards, such as high speed packet access (HSPA) and long-term evolution (LTE), especially for indoors coverage wherein conventional macrocell networks undergoes high penetration loss. One promising emerging technology that can improve indoors coverage is the use of small cells (also known as femtocells), which can be deployed at designated places to boost downlink capacity and coverage.

The co-existence of femtocells and original macrocells creates a two-tier cellular network, which results in many technical challenges. One of the most significant challenges is the increase in the observed interference when the cellular resources can be used unconditionally at any serving station. This creates new cross-tier interference between the two tiers as well as amplified co-tier interference inside each tier [1].

A possible solution for interference amplification is to allocate orthogonal subcarriers, as in orthogonal frequency division multiplexing (OFDM) system, for macrocells and femtocells, which in turns minimize the effect of cross-tier interference but at the expense of inefficient use of resources.

For the case when the subcarriers are simultaneously used in macrocells and femtocells, a power control method and resource allocation for the pilot and data signals can be useful to mitigate the effect of interference [2], but this requires extensive processing load. On the other hand, the access control scheme at femtocells can be a promoting technique to significantly reduce the effect of interference. There are three known types of access control schemes at femtocells, which are the closed, open, and hybrid access schemes [3]. For closed access femtocells, only authorized users are granted the access to femtocell resources, whereas the open access femtocells can serve any macrocell user within its spatial coverage. Moreover, the hybrid access femtocells can effectively manage the available resources to overcome the drawbacks of the closed and open access schemes [5]. Therefore, this paper focuses on the hybrid access femtocells mode of operation with the objective to effectively alleviate effect of amplified interference.

The use of multiple input multiple output (MIMO) arrays in wireless systems has been an attractive approach to substantially improve the diversity gain and/or the multiplexing gain. Among the techniques that gain benefits of MIMO links while maintaining reasonable complexity is the transmit antenna selection (TAS), which can reduce the number of radio frequency (RF) chains while achieving full diversity [4]. In point-to-point MIMO systems, TAS with receive maximal ratio combining (MRC) has been shown to achieve full transmit/receive diversity [6]. Therefore, this paper extends the use of MIMO TAS/MRC to overlaid two-tier cellular networks in order to precisely understand its benefits under new operation environment and design restrictions.

For irregular 2-D distributions of access points, such as in cognitive radio systems and heterogeneous networks, stochastic geometry can be useful to provide nodes spatial models, and hence, to characterize various interference sources distribution according to Poisson point process (PPP) [7]. In [8, 9], PPP was used to model interference from macrocell base stations. Moreover, it was adopted in [10, 11] to model cross-tier interference from femtocells.

The contributions of this paper are summarized as follows. We propose new approaches to characterize the impact of hybrid-access femtocells on the achieved performance of MIMO two-tier cellular networks with TAS/MRC in the presence of random PPP interference distributions. To fully understand the effect of MIMO spatial domain and interference distributions, two interference models are treated, in which interference sources are either fully-correlated or uncorrelated as observed at transmit array of a serving access point in any tier. The analysis presents new analytical formulations for the adopted system model, which are then used to provide novel analytical analysis for downlink performance at a user of interest.

The remainder of this paper is organized as follows. Section II discusses the system and interference models, and Section III describes in details the hybrid control access scheme at femtocells, and explains the relation between MIMO spatial domain and the observed interference. The exact and asymptotic performance of the MIMO SNR-based TAS/MRC under various interference scenarios is presented in Section IV. In Section V, some numerical examples, which are verified by simulations, are discussed, and finally, concluding remarks are provided in Section VI.

Mathematical Operators : We use $E\{.\}$ to denote the expectation operator. $f_x(.)$ denotes the Probability density function (PDF) of the random variable x. $\mathcal{F}_x(\gamma) = \Pr(x < \gamma)$ is the Cumulative Distribution Function (CDF) of random variable x. $\mathcal{M}_x(s) = \mathbb{E}\{e^{-sx}\}$ is the moment generating function for the random variable x. $\mathcal{L}^{-1}\{\}$ denotes the inverse Laplace transform. $H^{m,n}_{p,q}\left[Z\Big|^{(a_p,A_p)}_{(b_q,B_q)}\right]$ refers to the Fox H-function [12]. $\Gamma(.)$ is the complete gamma function.

II. System and Interference Models

A. System Model

A two-tier cellular network is considered, which contains two types of access points (or base stations). Throughout the remaining discussions, the notation $t = \{\mathrm{M}_B, \mathrm{F}_B\}$ is used to denote the macrocell based station or the femtocell access point, respectively. Moreover, $k = \{\mathrm{M}_U, \mathrm{F}_U\}$ is used to refer to the macrocell user or femtocell user, respectively.

The macrocell base station M_B can serve arbitrary number of macrocell users in its coverage area A_{M_B}. The area A_{M_B} is overlaid by arbitrary number of femtocell access points, which can provide service to an arbitrary number of femtocell users as well as controlled number of macrocell users (according to the proposed hybrid access at each femtocell access point F_B) in their coverage area A_{F_B}. The focus of this work is on orthogonal spectrum sharing scenario between M_B and F_B, which results in considering only the co-tier interference, while avoiding cross-tier interference.

B. Signal Model and SINR Analysis

In general, the nodes distributions (access points and users) are assumed to follow stationary Poisson Point Processes (PPPs), where the PPP Φ_ν, with $\nu = \{t, k\}$, has density λ_ν with an average number of nodes $\lambda_\nu A_\nu$ in area A_ν. An access point is assumed to have $N_t = \{N_{\mathrm{M}_B}, N_{\mathrm{F}_B}\}$ transmit antennas, whereas a user receive station is assumed to have $L_k = \{L_{\mathrm{M}_U}, L_{\mathrm{F}_U}\}$ receive antennas. The channel gains matrix $\mathbf{H}_{t,k}$ between access point t and user k has $N_t \times L_k$ complex-valued channel gain entries with $h_{t,i,k,r}$ denotes channel gain that is observed on the $t \to k$ link between the ith transmit antenna and the rth receive antenna, which can be expressed as

$$h_{t,i,k,r} = \alpha_{t,i,k,r} e^{-j\phi_{t,i,k,r}} \|d_{t,k}\|^{-v}, \tag{1}$$

where $\alpha_{t,i,k,r}$ and $\phi_{t,i,k,r}$ are the small-scale fading channel envelope and phase, respectively, with $\mathbb{E}\{\alpha^2_{t,i,k,r}\} = 1$ (unit mean fading power), v refers to the power path loss exponent, and $d_{t,k}$ is the separation distance of the $t \to k$ link, and $\| \cdot \|$ refers to the norm operation. The small-scale fading channel envelopes on MIMO links are modelled statistically independent and identical (i.i.d.) Rayleigh processes.

The composite received signal at user $k = \{\mathrm{M}_U, \mathrm{F}_U\}$ receive array that has a size L_k, when the ith transmit antenna at access point with $t = \{\mathrm{M}_B, \mathrm{F}_B\}$ is used, can be written as

$$\mathbf{z}_{t,i,k} = \mathbf{h}_{t,i,k} x_{t,i,k} + \sum_{m \in \Phi_t} x_{t,i,m} \mathbf{g}_{t,i,k,m} + \mathbf{n}_{t,i,k}, \tag{2}$$

where $\mathbf{z}_{t,i,k} = [z_{t,i,k,1}\, z_{t,i,k,2} \cdots z_{t,i,k,L_k}]^T$ is the column vector of size $L_k \times 1$ that contains received signal replicas at the receive antennas, $\mathbf{h}_{t,i,k} = [h_{t,i,k,1}\, h_{t,i,k,2} \cdots h_{t,i,k,L_k}]^T$ is the column vector of i.i.d. channel gains associated with desired data symbol, $\mathbf{g}_{t,i,k,m} = [g_{t,i,k,1,m}\, g_{t,i,k,2,m} \cdots g_{t,i,k,L_k,m}]^T$ is the column vector of the mth source of interference channel gains that is observed at the receive antennas, $\mathbf{n}_{t,i,k} = [n_{t,i,k,1}\, n_{t,i,k,2} \cdots n_{t,i,k,L_k}]^T$ is the column vector of i.i.d. AWGN variants, where each of these elements has zero mean and an average power of $\mathbb{E}\{|n_{t,i,k,r}|^2\} = \sigma_n^2$, the symbol T refers to vector transpose operation, $x_{t,i,k}$ is the transmitted data symbol from the ith transmit antenna whose power is $\mathbb{E}\{|x_{t,i,k}|^2\} = P_t/N_t$, and $x_{t,i,m}$ is the data symbol associated with the mth interference source that is by $t = \{\mathrm{M}_B, \mathrm{F}_B\}$ access point and affect terminal k with power $\mathbb{E}\{|x_{t,i,m}|^2\} = P_{t,i,m}$. The interference channel gain $g_{t,i,k,r,m}$ can be expressed as

$$g_{t,i,k,r,m} = \alpha_{t,i,k,r,m} e^{-j\phi_{t,i,k,r,m}} \|d_{t,k,m}\|^{-v}, \tag{3}$$

where $\alpha_{t,i,k,r,m}$ and $\phi_{t,i,k,r,m}$ are the channel fading envelope and phase, respectively. The fading envelope follows Rayleigh distribution with $\mathbb{E}\{\alpha^2_{t,i,k,r,m}\} = 1$, and $d_{t,k,m}$ is the separation distance between the mth interference source and the receive station k.

The *Pre*-TAS SINR following receive MRC (after weighting (2) by $\widehat{\mathbf{h}}^H_{t,i,k}/\|\widehat{\mathbf{h}}_{t,i,k}\|$, where $\widehat{\mathbf{h}}_{t,i,k} = \mathbf{h}_{t,i,k}/\|d_{t,k}\|^{-v}$, and the symbol H refers to conjugate transpose operation) of the link between $t = \{\mathrm{M}_B, \mathrm{F}_B\}$ and $k = \{\mathrm{M}_U, \mathrm{F}_U\}$ when the ith transmit antenna is used can be written as shown in (4), where $\overline{\gamma}_{D,t} \triangleq \frac{P_t/N_t}{\sigma_n^2}$ is the average SNR per receive antenna, $\overline{\gamma}_{I,i,t} \triangleq \frac{P_{t,i,m}}{\sigma_n^2}$ is the average INR per interferer on the ith transmit antenna, $\sum_{r=1}^{L_k} \alpha^2_{t,i,k,r} = \widehat{\mathbf{h}}_{t,i,\mathrm{M}_U} \widehat{\mathbf{h}}^H_{t,i,\mathrm{M}_U}$ that is a sum of i.i.d. exponential random variables each of which has unit mean, $\widetilde{\alpha}^2_{t,i,k,m} = |\widehat{\mathbf{h}}^H_{t,i,k} \widehat{\mathbf{g}}_{t,i,k}/\|\widehat{\mathbf{h}}_{t,i,k}\||^2$ follows an exponential

$$\gamma_{\text{SINR},t,i,k} = \frac{\frac{P_t}{N_t}\|d_{t,k}\|^{-v}\left|\frac{\widehat{\mathbf{h}}_{t,i,k}^H\widehat{\mathbf{h}}_{t,i,k}}{\|\widehat{\mathbf{h}}_{t,i,k}\|}\right|^2}{\sum_{m\in\Phi_t}P_{t,i,m}\|d_{t,k,m}\|^{-v}\left|\frac{\widehat{\mathbf{h}}_{t,i,k}^H\widehat{\mathbf{g}}_{t,i,k}}{\|\widehat{\mathbf{h}}_{t,i,k}\|}\right|^2+\left|\frac{\widehat{\mathbf{h}}_{t,i,k}^H\mathbf{n}_{t,i,k}}{\|\widehat{\mathbf{h}}_{t,i,k}\|}\right|^2} = \frac{\overline{\gamma}_{D,t}\|d_{t,k}\|^{-v}\sum_{r=1}^{L_k}\alpha_{t,i,k,r}^2}{\underbrace{\sum_{m\in\Phi_t}\overline{\gamma}_{I,i,t}\|d_{t,k,m}\|^{-v}\widetilde{\alpha}_{t,i,k,m}^2}_{I_{t,i,k}}+1}. \tag{4}$$

distribution of unit mean, and $|\widehat{\mathbf{h}}_{t,i,k}^H\mathbf{n}_{t,i,k}/\|\widehat{\mathbf{h}}_{t,i,k}\||^2$ is an AWGN distributed variant of zero mean and an average power σ_n^2.

By exploiting the statistical independence between small-scale fading and point processes [13], the aggregate interference power in (4), which is denoted by $I_{t,i,k} = \sum_{m\in\Phi_t}\overline{\gamma}_{I,i,t}\|d_{t,k,m}\|^{-v}\alpha_{t,i,k,m}^2$, has a Laplace transform, $\psi_{I_{t,i,k}}(s) = \mathbb{E}\{e^{-sI_{t,i,k}}\}$, that is expressed as

$$\begin{aligned}\psi_{I_{t,i,k}}(s) &= \mathbb{E}\left\{\prod_{m\in\Phi_t}\mathbb{E}\left\{e^{-s(\overline{\gamma}_{I,i,t}\|d_{t,k,m}\|^{-v}\alpha_{t,i,k,m}^2)}\right\}\right\},\\ &= \exp\left[-\lambda_t\int_{R^2}\left(1-\frac{1}{1+s\overline{\gamma}_{I,t}z^{-v}}\right)dz\right],\\ &= \exp\left[-\lambda_t(s\overline{\gamma}_{I,i,t})^{\frac{2}{v}}\beta(v)\right],\ v>2,\end{aligned} \tag{5}$$

where $\beta(v) = \frac{2\pi^2}{v\sin(2\pi/v)}$. Note that $I_{t,i,k}$ follows alpha stable distribution with characteristic exponent $\frac{2}{v}$ and dispersion $\lambda_t(\overline{\gamma}_{I,i,t})^{\frac{2}{v}}\beta(v)$.

Note that the resulting interference $I_{t,i,k}$ that is observed at $k = \{\mathrm{M}_U, \mathrm{F}_U\}$ may be function of the index of the ith antenna associated with $t = \{\mathrm{M}_B, \mathrm{F}_B\}$. This is realized if the transmit antennas at $t = \{\mathrm{M}_B, \mathrm{F}_B\}$ are perfectly separated through, for instance, orthogonal codes. In this case, the interference observed from $t = \{\mathrm{M}_B, \mathrm{F}_B\}$ network undergoes perfect spatial de-correlation versus the used transmit antenna. When the transmit antennas are not separated, it is reasonable to assume that $I_{t,i,k} = I_{t,k}$, for $i = 1, 2, \ldots, N_t$, which implies that the interference observed from $t = \{\mathrm{M}_B, \mathrm{F}_B\}$ network undergoes high spatial correlation, regardless of the used transmit antenna at the serving station. These two cases are treated in the context of TAS scheme in the following part.

C. Post-TAS SINR

The selection of the suitable transmit antenna at the serving station t can be implemented to maximize the desired combined SNR after MRC (i.e, SNR-based TAS) at the receive station. For the case when the transmit antennas are perfectly separated, it follows that the SNR-based TAS results in the *Post*-TAS SINR (i.e, the combined SINR after TAS), $\gamma_{\text{SINR},t,k}$, that is given $\gamma_{\text{SINR},t,k} = \frac{\max_{i=1,\ldots,N_t}\{\gamma_{\text{SNR},t,i,k}\}}{I_{t,k}+1}$, where $\gamma_{\text{SNR},t,i,k} = \overline{\gamma}'_{D,t}\sum_{r=1}^{L_k}\alpha_{t,i,k,r}^2$ is the combined *Pre*-TAS SNR, $\overline{\gamma}'_{D,t} = \overline{\gamma}_{D,t}\|d_{t,k}\|^{-v}$, and $I_{t,k} = I_{t,i,k}$ if the ith transmit antenna is selected. On the other hand, when the transmit antennas are not separated, it follows that $\gamma_{\text{SINR},t,k}$ has the same form but with $I_{t,k} = I_{t,i,k}$ for any $i = 1, \ldots, N_t$.

III. Hybrid Access Scheme

For the sake of simplicity, the following analysis considers $k = \mathrm{M}_U$ as being the user of interest. However, in light of the preceding results in the previous section, the analysis can be modified directly to treat $k = \mathrm{F}_U$. Following footsteps in [5], the selection of the suitable downlink (or serving access point t) for the M_U can be made according to

$$\rho = \frac{\eta_0\sum_{i=1}^{N_{\mathrm{M}_B}} I_{\mathrm{M}_B,i,\mathrm{M}_U}+\sigma_n^2}{\sum_{i=1}^{N_{\mathrm{F}_B}} I_{\mathrm{F}_B,i,\mathrm{M}_U}+\sigma_n^2}. \tag{6}$$

The identification of the suitable serving access point is related to the amount of interference power plus noise at M_U. The number of femtocell access points $\Phi_{\mathrm{F}_B}(A)$ in close vicinity of M_U over an area coverage A follows a Poisson distribution with mean value of $\lambda_{\mathrm{F}_B}A$, and number of femtocell users $\Phi_{\mathrm{F}_U}(A)$ in the same area follows also a Poisson distribution with mean value of $\lambda_{\mathrm{F}_U}A$. The condition to allow for a hybrid access for an M_U is given as

$$\eta_0 = \sum_{x=0}^{\infty}\left[\sum_{n=0}^{x-1}e^{-\lambda_{\mathrm{F}_U}A}\frac{(\lambda_{\mathrm{F}_U}A)^n}{n!}\right]e^{-\lambda_{\mathrm{F}_B}A}\frac{(\lambda_{\mathrm{F}_B}A)^x}{x!}, \tag{7}$$

where F_B is assumed to serve an emerging user at a time. If $\rho < 1$, then M_U selects M_B to receive downlink service. Otherwise, M_U can be served an F_B. The probability of the event $\rho < 1$ can be expressed as

$$\Pr\{\rho<1\} = \Pr\left\{\frac{\eta_0\sum_{i=1}^{N_{\mathrm{M}_B}} I_{\mathrm{M}_B,i}+\sigma_n^2}{\sum_{i=1}^{N_{\mathrm{F}_B}} I_{\mathrm{F}_B,i}+\sigma_n^2}<1\right\}, \tag{8}$$

where the subscript M_U in $I_{\mathrm{M}_B,i,\mathrm{M}_U}$ and $I_{\mathrm{M}_B,i,\mathrm{M}_U}$ in ρ expression is dropped in the preceding equation for simplicity.

A. Case of Identical Interference on Transmit Antennas

For the case when $I_{\mathrm{M}_B,i} = I_{\mathrm{M}_B}$ and $I_{\mathrm{F}_B,i} = I_{\mathrm{F}_B}$, regardless of the transmit antenna index, it follows that

$$\Pr\{\rho_1<1\} = \Pr\{\eta I_{\mathrm{M}_B} - I_{\mathrm{F}_B} < 0\}, \tag{9}$$

where $\eta = \eta_0 N_{\mathrm{M}_B}/N_{\mathrm{F}_B}$. To proceed with the analysis, the Laplace transform of I_{M_B} (or I_{F_B}) that follows the form in (5), can be re-written as

$$\begin{aligned}\psi_{I_t}(s) &= \exp\left[-\lambda_t(s\overline{\gamma}_{I,t})^{\frac{2}{v}}\beta(v)\right]\\ &\overset{(a)}{=} H_{0,1}^{1,0}\left[\lambda_t(s\overline{\gamma}_{I,t})^{\frac{2}{v}}\beta(v)\Big|_{(0,1)}^{-}\right],\end{aligned} \tag{10}$$

where $\overline{\gamma}_{I,t} = \overline{\gamma}_{I,t,i,\mathrm{M}_U}$ in this case, and (a) follows from the definition of Fox H-function [12], [13, eq. 9]. The PDF of I_t

can be then obtained by taking the inverse Laplace transform of (10) to give [14, eq. 7]

$$f_{I_t}(y)=\frac{1}{j2\pi}\lim_{T\to\infty}\int_{\epsilon-jT}^{\epsilon+jT}e^{sy}\psi_{I_t}(s)ds,$$
$$=\frac{1}{y}H_{1,1}^{1,0}\left[\lambda_t(y\overline{\gamma}_{I,t})^{\frac{2}{v}}\,\beta(v)\Big|_{(0,1)}^{(0,\frac{v}{2})}\right]. \tag{11}$$

Moreover, the CDF of I_t can now be obtained as [14, eq. 15]

$$F_{I_t}(x)=\int_0^x f_{I_t}(y)dy=H_{1,1}^{1,0}\left[\lambda_t(x\overline{\gamma}_{I,t})^{\frac{2}{v}}\,\beta(v)\Big|_{(0,1)}^{(1,\frac{v}{2})}\right]. \tag{12}$$

Using the preceding results, the expression for $\Pr\{\rho_1<1\}=\Pr\{\eta I_{\mathrm{M}_B}-I_{\mathrm{F}_B}<0\}$ is expressed using [15, eq. 4.12] as

$$\Pr\{\rho_1<1\}=\int_0^{+\infty}F_{I_{\mathrm{M}_B}}\left(\frac{x}{\eta}\right)f_{I_{\mathrm{F}_B}}(x)dx,$$
$$=\frac{v}{2}H_{2,2}^{1,1}\left[\frac{\lambda_{\mathrm{M}_B}}{\lambda_{\mathrm{F}_B}}\left(\frac{\overline{\gamma}_{I,\mathrm{M}_B}}{\eta\overline{\gamma}_{I,\mathrm{F}_B}}\right)^{\frac{2}{v}}\Big|_{(0,1),(1,\frac{2}{v})}^{(1,1),(1,\frac{2}{v})}\right]. \tag{13}$$

B. Case of Independent Interference on Transmit Antennas

For the case when $I_{\mathrm{M}_B,i}$ and $I_{\mathrm{F}_B,i}$ de-correlate with the index of the transmit antenna, and based on (8), we have

$$\Pr\{\rho_2<1\}=\int_0^{\infty}F_{\sum_{i=1}^{N_{\mathrm{M}_B}}I_{\mathrm{M}_B,i}}\left(\frac{x}{\eta_0}\right)f_{\sum_{i=1}^{N_{\mathrm{F}_B}}I_{\mathrm{F}_B,i}}(x)dx, \tag{14}$$

where

$$\psi_{\sum_{i=1}^{N_t}I_{t,i}}(s)=\prod_{i=1}^{N_t}\exp\left[-\lambda_t(s\overline{\gamma}_{I,t,i})^{\frac{2}{v}}\,\beta(v)\right],$$
$$=H_{0,1}^{1,0}\left[\lambda_t s^{\frac{2}{v}}\left(\sum_{i=1}^{N_t}(\overline{\gamma}_{I,t,i})^{\frac{2}{v}}\right)\beta(v)\Big|_{(0,1)}^{-}\right] \tag{15}$$

Then it can be shown that the PDF and CDF of $\sum_{i=1}^{N_t}I_{t,i}$ have similar forms to those given in (11) and (12), respectively, but with the parameter $(\overline{\gamma}_{I,t})^{\frac{2}{v}}$ therein is replaced by $\sum_{i=1}^{N_t}(\overline{\gamma}_{I,t,i})^{\frac{2}{v}}$. Therefore, the result for $\Pr\{\rho_2<1\}$ in (14) is expressed as

$$\Pr\{\rho_2<1\}=\frac{v}{2}H_{2,2}^{1,1}\left[\frac{\lambda_{\mathrm{M}_B}}{\lambda_{\mathrm{F}_B}}\frac{\sum_{i=1}^{N_t}(\overline{\gamma}_{I,\mathrm{M}_B,i})^{\frac{2}{v}}}{(\eta_0)^{\frac{2}{v}}\sum_{i=1}^{N_t}(\overline{\gamma}_{I,\mathrm{F}_B,i})^{\frac{2}{v}}}\Big|_{(0,1),(1,\frac{2}{v})}^{(1,1),(1,\frac{2}{v})}\right]. \tag{16}$$

Note that the result for $\Pr\{\rho_2<1\}$ in (16) becomes identical to that for $\Pr\{\rho_1<1\}$ in (13) when $\overline{\gamma}_{I,t,i}=\overline{\gamma}_{I,t}$, for $i=1,2,\ldots,N_t$.

IV. Outage performance Analysis with SNR-Based *Post*-TAS

A. Exact Analysis

The outage probability is defined as the probability that the combined SINR at M_U falls below a predetermined threshold, γ_{th}; $\mathrm{P}_{\text{out}}=\Pr\{\gamma_{\text{SINR}}<\gamma_{th}\}$. The following parts presents the outage probability for the two cases of interference among transmit antennas.

1) Case of Identical Interference on Transmit Antennas: The outage probability in this case can be expressed as

$$\mathrm{P}_{\text{out}}=\Pr\{\rho_1<1\}\Pr\{\gamma_{\text{SINR},\mathrm{M}_B,\mathrm{M}_U}<\gamma_{th}\}$$
$$+(1-\Pr\{\rho_1<1\})\Pr\{\gamma_{\text{SINR},\mathrm{F}_B,\mathrm{M}_U}<\gamma_{th}\}, \tag{17}$$

where $\Pr\{\rho_1<1\}$ is given in (13) and $\gamma_{\text{SINR},t,k}$ is defined with $I_{t,k}=I_{t,i,k}$, regardless of the index of the selected transmit antenna. The term $\Pr\{\gamma_{\text{SINR},t,k}<\gamma_{th}\}$ in (17) for $t=\{\mathrm{M}_B,\mathrm{F}_B\}$ and $k=\mathrm{M}_U$ can be expressed as $\Pr\{\gamma_{\text{SINR},t,k}<\gamma_{th}\}=\int_0^\infty F_X((1+y)\gamma_{th})f_{I_{t,k}}(y)dy$, where $X=\max_{i=1,\ldots,N_t}\{\gamma_{\text{SNR},t,i,k}\}$ and $\gamma_{\text{SNR},t,i,k}=\overline{\gamma}'_{D,t}\sum_{r=1}^{L_k}\alpha_{t,i,k,r}^2$. Then $\gamma_{\text{SNR},t,i,k}$ has a Gamma distribution with parameters $\gamma_{\text{SNR},t,i,k}\sim\mathcal{G}\left(L_k,1/\overline{\gamma}'_{D,t}\right)$. Therefore the CDF of X can be expressed as

$$F_X(x)=\prod_{i=1}^{N_t}\frac{1}{\Gamma(L_k)}\gamma\left(L_k,\frac{x}{\overline{\gamma}'_{D,t}}\right), \tag{18}$$

where $\gamma(a,x)$ is the lower incomplete Gamma function. Since L_k is an integer, and using the finite summation representation of $\gamma(a,x)$, the result in (18) can be expanded as

$$F_X(x)=\left[1-e^{-(x/\overline{\gamma}'_{D,t})}\sum_{r=1}^{L_k}\frac{\left(\frac{x}{\overline{\gamma}'_{D,t}}\right)^{r-1}}{(r-1)!}\right]^{N_t},$$
$$\overset{(a)}{=}1-\sum_{n=1}^{N_t}\binom{N_t}{n}(-1)^{n-1}e^{-(nx/\overline{\gamma}'_{D,t})}$$
$$\times\sum_{r_1=1}^{L_k}\sum_{r_2=1}^{L_k}\cdots\sum_{r_n=1}^{L_k}\frac{\left(\frac{x}{\overline{\gamma}'_{D,t}}\right)^{r_{\text{sum}}}}{\prod\limits_{p=1}^{n}(r_p-1)!}, \tag{19}$$

where $\overset{(a)}{=}$ follows from Binomial expansion and $r_{\text{sum}}=\sum_{p=1}^{n}(r_p-1)$. Inserting (19) into $\Pr\{\gamma_{\text{SINR},t,k}<\gamma_{th}\}$ gives

$$\Pr\{\gamma_{\text{SINR},t,k}<\gamma_{th}\}=1-\sum_{n=1}^{N_t}\binom{N_t}{n}(-1)^{n-1}e^{-(n\gamma_{th}/\overline{\gamma}'_{D,t})}$$
$$\times\sum_{r_1=1}^{L_k}\sum_{r_2=1}^{L_k}\sum_{r_3=1}^{L_k}\cdots\sum_{r_n=1}^{L_k}\frac{\left(\frac{\gamma_{th}}{\overline{\gamma}'_{D,t}}\right)^{r_{\text{sum}}}}{\prod\limits_{p=1}^{n}(r_p-1)!}\sum_{j=0}^{r_{\text{sum}}}\binom{r_{\text{sum}}}{j}\mathcal{J}_1, \tag{20}$$

where $\mathcal{J}_1$ can be evaluated as

$$\mathcal{J}_1=\int_0^\infty y^j\,e^{-\left(n\gamma_{th}/\overline{\gamma}'_{D,t}\right)y}f_{I_{t,k}}(y)dy,$$
$$\overset{(a)}{=}(-1)^j\left[\frac{d^j\psi_{I_{t,k}}(s)}{ds^j}\right]\Big|_{s=(n\gamma_{th}/\overline{\gamma}'_{D,t})},$$
$$\overset{(b)}{=}(-1)^j\left(\frac{n\gamma_{th}}{\overline{\gamma}'_{D,t}}\right)^{-j}H_{1,2}^{2,0}\left[\alpha_t\left(\frac{n\gamma_{th}}{\overline{\gamma}'_{D,t}}\right)^{\frac{2}{v}}\Big|_{(j,\frac{2}{v}),(0,1)}^{(0,\frac{2}{v})}\right], \tag{21}$$

where $\alpha_t = \lambda_t \beta(v)\overline{\gamma}_{I,t}^{\frac{2}{v}}$, (a) follows from known Laplace transform property, and (b) follows from the derivative property of Fox H-function [12, eq.2.2.2].

Substituting by $\mathcal{J}_1$ into (20) and then inserting the resulting expression of $\Pr\{\gamma_{\text{SINR},t,k} < \gamma_{th}\}$ for $t = \{\mathrm{M}_B, \mathrm{F}_B\}$ and $k = \mathrm{M}_U$ in (17) yields the desired final results for the outage probability performance of M_U.

2) Case of Independent Interference on Transmit Antennas: The outage probability in this case can be expressed as

$$\begin{aligned}\mathrm{P}_{\text{out}} &= \Pr\{\rho_2 < 1\}\Pr\{\gamma_{\text{SINR},\mathrm{M}_B,\mathrm{M}_U} < \gamma_{th}\}\\ &\quad + (1 - \Pr\{\rho_2 < 1\})\Pr\{\gamma_{\text{SINR},\mathrm{F}_B,\mathrm{M}_U} < \gamma_{th}\},\end{aligned} \tag{22}$$

where $\Pr\{\rho_2 < 1\}$ is given in (16) and $\gamma_{\text{SINR},t,k}$ is defined with $I_{t,k} = I_{t,i,k}$ only if the ith transmit antenna is selected. The term $\Pr\{\gamma_{\text{SINR},t,k} < \gamma_{th}\}$ in (22) for $t = \{\mathrm{M}_B, \mathrm{F}_B\}$ and $k = \mathrm{M}_U$ can be expressed as

$$\Pr\{\gamma_{\text{SINR},t,k} < \gamma_{th}\} = \sum_{i=1}^{N_t} p_i \int_0^\infty F_X((1+y)\gamma_{th}) f_{I_{t,i,k}}(y)dy, \tag{23}$$

where $f_{I_{t,i,k}}(x)$ is the PDF of $I_{t,i,k}$, which can be obtained as $f_{I_{t,i,k}}(x) = \frac{1}{\Gamma(L_k)}\left(\frac{1}{\overline{\gamma}'_{D,t}}\right)^{L_k} x^{L_k-1} e^{-x/\overline{\gamma}'_{D,t}}$, and the term X has the same CDF that is given in (19). Moreover, the term p_i in (23) denotes the probability that the ith transmit antenna is selected and hence the associated interference power $I_{t,i,k}$ is observed. This term can be obtained as the probability of the following event: $\gamma_{\text{SNR},t,i,k} > \gamma_{\text{SNR},t,q,k}$, for $q = 1, 2, \ldots, N_t$ and $q \neq i$. Then it can be written that

$$p_i = \int_0^\infty \left[1 - e^{-\left(\frac{x}{\overline{\gamma}'_{D,t}}\right)} \sum_{r=1}^{L_k} \frac{\left(\frac{x}{\overline{\gamma}'_{D,t}}\right)^{r-1}}{(r-1)!}\right]^{N_t-1} f_{\gamma_{\text{SNR},t,i,k}}(x)dx. \tag{24}$$

Expanding the term between brackets in (24) gives

$$p_i = 1 - \sum_{n=1}^{N_t-1} \binom{N_t}{n} (-1)^{n-1} \sum_{r_1=1}^{L_k}\sum_{r_2=1}^{L_k} \cdots \sum_{r_n=1}^{L_k} \frac{\mathcal{J}_2}{\prod_{p=1}^{n}(r_p-1)!},$$

where

$$\begin{aligned}\mathcal{J}_2 &= \frac{\left(\frac{1}{\overline{\gamma}'_{D,t}}\right)^{r_{\text{sum}}+L_k}}{\Gamma(L_k)} \int_0^\infty e^{-(x(n+1)/\overline{\gamma}'_{D,t})} x^{r_{\text{sum}}+L_k-1} dx,\\ &= \frac{\Gamma(r_{\text{sum}}+L_k)}{\Gamma(L_k)} \left(\frac{1}{n+1}\right)^{r_{\text{sum}}+L_k}.\end{aligned} \tag{25}$$

The final result has a form that is similar to that given in (20) but with $\mathcal{J}_1$ therein is replaced by

$$\mathcal{J}_3 = (-1)^j \left(\frac{n\gamma_{th}}{\overline{\gamma}'_{D,t}}\right)^{-j} H_{1,2}^{2,0}\left[\phi_t \left(\frac{n\gamma_{th}}{\overline{\gamma}'_{D,t}}\right)^{\frac{2}{v}} \middle|\begin{matrix}(0,\frac{2}{v})\\ (j,\frac{2}{v}),(0,1)\end{matrix}\right] \tag{26}$$

Substituting by $\mathcal{J}_3$ into (23) and then inserting the resulting expression of $\Pr\{\gamma_{\text{SINR},t,k} < \gamma_{th}\}$ for $t = \{\mathrm{M}_B, \mathrm{F}_B\}$ and $k = \mathrm{M}_U$ in (22) yields the desired results for the outage probability of M_U.

B. Asymptotic Analysis

The outage probability at high SNR regime is evaluated using the Taylor series expansion of exponentials ($e^{-x} \approx 1 - x$) as $x \to 0$ [16, eq.1.211.1] and the expansion of H-function at $z = 0$ [12, eq.1.8.1].

1) Case of Identical Interference on Transmit Antennas: The asymptotic outage probability is written for (20) as

$$\mathrm{P}_{t,k}^{\infty} = (G_c \overline{\gamma}'_{D,t})^{-G_d} + o(\overline{\gamma}'_{D,t}{}^{-G_d}), \tag{27}$$

$$\approx \frac{\alpha_t}{(L_k!)^{N_t}} \left(\frac{\gamma_{th}}{\overline{\gamma}'_{D,t}}\right)^{\min(\frac{2}{v}, L_k N_t)}, \tag{28}$$

where $o(.)$ is expressed as $f(x) = o(g(x))$ as $x \to x_0$ if $\lim_{x\to x_0} \frac{f(x)}{g(x)} = 0$. G_d is the diversity gain and is defined as the slope of the asymptotic curve which is given by $G_d = \lim_{\overline{\gamma}'_{D,t}\to\infty} \frac{-\log \Pr\{\gamma_{\text{SINR},t,k} < \gamma_{th}\}}{\log \overline{\gamma}'_{D,t}}$. G_c is the coding gain representing the SNR advantage of the asymptotic curve relative to a reference $\overline{\gamma}'_{D,t}{}^{-G_c}$. Comparing expressions in equations (27), and (28), the achievable diversity and coding gain are given respectively by $G_a^{\text{Id}} = \min\left(\frac{2}{v}, L_k N_t\right)$ and, $G_c^{\text{Id}} = \left(\frac{\alpha_t}{\overline{\gamma}'_{D,t}(L_k!)^{N_t}}\right)^{-\frac{1}{\min(\frac{2}{v}, L_k N_t)}}$. This concludes that the outage probability performance is dominated by the effect of the path loss of the interference sources. Substituting the resulting expression of $\mathrm{P}_{t,k}^{\infty}$ in (28) for $t = \{\mathrm{M}_B, \mathrm{F}_B\}$ and $k = \mathrm{M}_U$ in (17) yields the desired final results for the asymptotic outage probability of M_U.

2) Case of Independent Interference on Transmit Antennas: The asymptotic outage probability can be obtained by using (20) and replacing $\mathcal{J}_1$ by $\mathcal{J}_3$ obtained as follows:

$$\mathrm{P}_{t,k}^{\infty} \approx \frac{\phi_t}{(L_k!)^{N_t}} \left(\frac{\gamma_{th}}{\overline{\gamma}'_{D,t}}\right)^{\min(\frac{2}{v}, L_k N_t)}. \tag{29}$$

The diversity and coding gains are expressed as $G_a^{\text{Ind}} = \min\left(\frac{2}{v}, L_k N_t\right)$, and $G_c^{\text{Ind}} = \left(\frac{\phi_t}{\overline{\gamma}'_{D,t}(L_k!)^{N_t}}\right)^{-\frac{1}{\min(\frac{2}{v}, L_k N_t)}}$. Substituting the resulting expression of $\mathrm{P}_{t,k}^{\infty}$ for $t = \{\mathrm{M}_B, \mathrm{F}_B\}$ and $k = \mathrm{M}_U$ in (22) yields the desired final results for the asymptotic outage probability performance of M_U. Similarly, the outage performance of this scenario is also dominated by the pass loss of the interference sources.

As can be observed from (28) and (29), the considered system under the two identical and independent interference schemes has the same diversity gain. Thus, we observe that the SINR gap between the two scenarios depend on the coding gain. Therefore, the SINR gap is characterised as

$$\frac{G_c^{\text{Id}}}{G_c^{\text{Ind}}} = \left(\frac{\alpha_t}{\phi_t}\right)^{-\frac{1}{\min(\frac{2}{v}, L_k N_t)}}. \tag{30}$$

This result indicates that for the same outage probability, the performance of identical interference scheme outperforms the independent interference sources on same transmit antennas by $10(1/\min(\frac{2}{v}, L_k N_t)) \log(\alpha_t/\phi_t)$ dB.

V. Numerical Results

In this section, we provide the numerical results to verify our analytical results derived for the hybrid access two tier cellular network. We consider the threshold SINR of $\gamma_{th} = 5$dB. All the obtained analytical results are verified using Monte Carlo simulations. The simulations are obtained by averaging over 10^5 independent trials.

Fig. 1 depicts the SNR-based selection outage probability for identical interference case versus the average SNR. In particular, the outage probability is performed at different selection probability values $\rho_1 = \{0.8, 0.6, 0.4\}$ and different value of α_t at selection probability $\rho_1 = 0.8$. The figure shows that the outage probability is enhanced when selection probability is decreased, i.e., the macro user tends to select the macro link when the interference is high on the femto link. In addition, it is obvious from the figure that increasing α_t degrades the outage probability.

Fig. 2 depicts the SNR-based selection outage probability performance for identical interference case versus the average SNR. The error probability is performed at both different number of transmit antennas $N_t = \{2, 3, 4\}$ and different number of receive antennas $L_k = \{2, 3, 4\}$, at selection probability $\rho_1 = 0.8$. It is observed that increasing the number of transmit antennas and number of receive antennas enhances the outage probability due to the diversity gained by selecting the signal level.

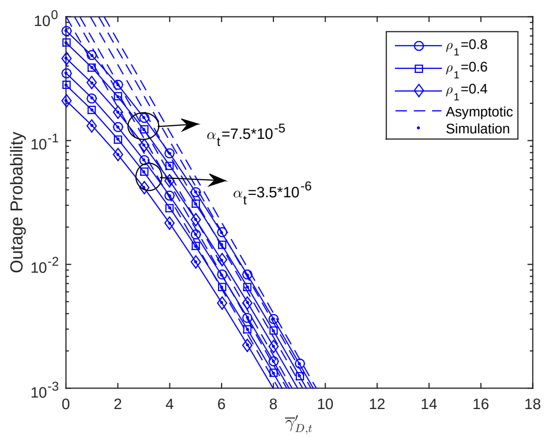

Fig. 1. SNR-based selection Outage probability at different selection probability $\rho_1 = \{0.8, 0.6, 0.4\}$..

VI. Conclusion

The paper has presented new approaches to characterize the performance of hybrid control-access small cells in MIMO cellular networks with random interference distributions. The analysis has considered MIMO transceivers with SNR-based TAS/MRC under different practical network-level models of interference sources. New analytical approaches that characterize the downlink outage probability performance have been developed, from which the outage performance in high signalto- noise (SNR) regime has been studied. The outcomes of this paper provided new system models that incorporate the hybrid access operation of femtocells, the random distributions

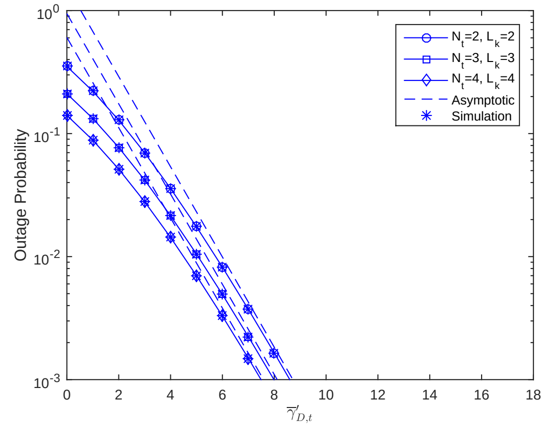

Fig. 2. SNR-based selection Outage probability at both different N_t transmit antennas and different receive antennas L_k.

interference sources, and the possible scenarios of interference models on MIMO spatial domain into the performance quantification of overlaid two-tier cellular networks.

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