Analysis and Enhancement of CSMA/CA With Deferral in Power-Line Communications

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Abstract—Power-line communications are employed in home networking to provide easy and high-throughput connectivity. The IEEE 1901, the MAC protocol for power-line networks, employs a CSMA/CA protocol similar to that of 802.11, but is substantially more complex, which probably explains why little is known about its performance. One of the key differences between the two protocols is that whereas 802.11 only reacts upon collisions, 1901 also reacts upon several consecutive transmissions and thus can potentially achieve better performance by avoiding unnecessary collisions. In this paper, we propose a model for the 1901 MAC. Our analysis reveals that the default configuration of 1901 does not fully exploit its potential and that its performance degrades with the number of stations. Based on analytical reasoning, we derive a configuration for the parameters of 1901 that drastically improves throughput and achieves optimal performance without requiring the knowledge of the number of stations in the network. In contrast, 802.11 requires knowing the number of contending stations to provide a similar performance, which is unfeasible for realistic traffic patterns. We confirm our results and enhancement with testbed measurements, by implementing the 1901 MAC protocol on WiFi hardware.

Index Terms—Power-line communications, HomePlug AV, IEEE 1901, CSMA/CA, enhancement.

I. INTRODUCTION

POWER-LINE communications (PLC) are developing rapidly. HomePlug, the leading alliance for PLC standardization, proposes different solutions for home automation and high data-rate local networks, with physical rates up to 1 Gbps. Even though 95% of PLC devices follow the HomePlug specification [2], the MAC layer of this specification has received little attention so far in the research community, in contrast to

the PHY layer. In particular, no work has investigated how far from optimality this MAC protocol is.

As PLC technology is becoming an important component in home networks, residential buildings are expected to host networks with a high number of PLC stations. These PLC stations interfere with each other, because – in contrast to wireless technologies that rely on different communication channels – PLC utilizes the entire available bandwidth (1.8–80 MHz) for communication. Therefore, there is need for enhancements at the MAC layer, as an efficient MAC is essential to maintain good performance when many stations contend for the medium. In this paper, we focus on understanding the MAC layer dynamics, and on enhancing its performance building on this understanding.

Due to the shared nature of power lines, HomePlug devices employ a multiple-access scheme based on CSMA/CA that is specified by the IEEE 1901 standard [3]. The 1901 CSMA/CA protocol bears some resemblance to the CSMA/CA mechanism employed by IEEE 802.11, which has been extensively studied in the literature (for instance, in [4] and [5]). Nevertheless, 1901 differs from 802.11 in that its CSMA/CA mechanism is more complex, making its theoretical analysis challenging. In particular, in addition to using a backoff counter, it also uses a so-called deferral counter. The deferral counter significantly increases the state-space required to describe the backoff procedure, which contrasts with the comparatively small state-space required to analyze 802.11 (see, e.g., the Markov chain used in [4]). As a result, the analysis of 1901 has received little attention despite the commercial success and massive adoption of PLC technologies.

From a general perspective, it turns out that 1901 implements an approach to contention resolution that differs drastically from the 802.11 CSMA/CA. In particular, while 802.11 can only react to contention (by doubling its contention window) after detecting a collision, 1901 can already react when it senses the medium busy during a certain number of time slots (given by the deferral counter). Such a protocol design has two distinct advantages over 802.11:

1) The contention window can be increased as many times as required to reach appropriate backoff durations without suffering any collision. In contrast, with 802.11 the contention window can only be doubled after a collision, and thus in many cases several collisions need to occur before the contention window reaches the appropriate value. As a result, 1901 can reduce the
channel time wasted in collisions, potentially leading to better performance.

2) By appropriately selecting the number of busy slots that trigger an increase of the contention window, we can adjust with fine granularity the level of contention that triggers a reaction. In contrast, this is not possible in 802.11, where contention is detected by the binary signal given by channel occupation: either the channel is busy upon a transmission attempt, which yields a collision, or it is not, and any finer refinements are not possible.

The above reasoning suggests that 1901 can substantially outperform 802.11 if properly configured. However, as we will observe, the default configuration of 1901 does not achieve the level of efficiency that one would expect given these premises. One important cause of the (relatively) poor performance of the protocol is the lack of an accurate and simple analysis that provides an insightful understanding of its dynamics and that can be used to configure the protocol appropriately. As we will show, performance can indeed be largely improved.

Motivated by the above, in this paper we propose a framework for modeling and enhancing the CSMA/CA process of 1901. The main contributions of this paper are the following. First, we introduce a model that accurately captures 1901 performance while reducing very substantially the required state-space; this model comes in the form of a fixed-point equation which we show that admits a unique solution. Second, we employ our model to compute a configuration that boosts throughput. Our configuration consists in simply setting existing MAC parameters to appropriate values, and thus can be readily implemented by manufacturers of PLC devices. The proposed configuration provides drastic performance improvements. Third, we validate our enhancement in a real testbed, by implementing the 1901 mechanism on WiFi hardware.

One of the most remarkable results of the paper is that, with the proposed configuration, 1901 provides a performance very close to that of an optimally configured MAC protocol without knowing the total number of contending stations $N$. In contrast, similar methods for enhancing the 802.11 CSMA/CA process do require knowing $N$ (see, e.g., [6]), which challenges their practicality in real deployments where $N$ varies in time. Thus, with our proposed configuration, the 1901 protocol represents an interesting step towards a practical and optimal MAC protocol.

The rest of this paper is organized as follows. In the next section, we present the 1901 backoff procedure. In Section III, we review related work on models and enhancements for MAC protocols. We then present our model for 1901 in Section IV, and propose enhancements for 1901 configuration in Section V. Our model and proposed configurations are evaluated in Section VI. Finally, we give our concluding remarks in Section VII.

II. THE 1901 MAC MECHANISM

In this section, we present the relevant aspects of the 1901 CSMA/CA procedure [3], giving insights on the requirements that drove the design of this intricate protocol.

The first PLC specification that included this CSMA/CA process is HomePlug 1.0, whose slot duration was determined by the time required by a station to detect a preamble transmission, which was equal to the duration of 7 symbols (i.e., $35.84 \mu s$) [7]. Although newer technologies have different symbol durations, the slot duration has remained the same for all HomePlug standards for backward compatibility. Observe that the slot duration is large compared to the one of 802.11 (which is $9 \mu s$ for 802.11a/g/n/ac). In the next paragraphs, we explain the effect of the slot duration in the backoff process.

The backoff process of 1901 uses two counters: the backoff counter ($BC$) and the deferral counter ($DC$).\(^1\) We now discuss the common features of 1901 and 802.11, and we elaborate later on the deferral counter. When a new packet arrives for transmission, the station starts at backoff stage 0, and it draws the backoff counter $BC$ uniformly at random in $[0, \ldots, CW_0 - 1]$, where $CW_0$ refers to the contention window used at backoff stage 0. Similarly to 802.11, $BC$ is decreased by 1 at each time slot if the station senses the medium to be idle, and it is frozen when the medium is sensed busy. In case the medium is sensed busy, $BC$ is also decreased by 1 once the medium is sensed idle again. When $BC$ reaches 0, the station attempts to transmit the packet. Also similarly to 802.11, the station jumps to the next backoff stage if the transmission fails (unless it is already at the last backoff stage, in which case it re-enters this backoff stage). When entering backoff stage $i$, a station draws $BC$ uniformly at random in $[0, \ldots, CW_i - 1]$, where $CW_i$ is the contention window at backoff stage $i$, and the process is repeated. For 802.11, the contention window is doubled between two successive backoff stages, and thus $CW_i = 2^i CW_0$. For 1901, $CW_i$ depends on the priority level, and is given in Table I.

<table>
<thead>
<tr>
<th>Priority class</th>
<th>CA0/CA1</th>
<th>CA2/CA3</th>
</tr>
</thead>
<tbody>
<tr>
<td>backoff stage $i$</td>
<td>$CW_i$</td>
<td>$d_i$</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>16</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>32</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>64</td>
<td>15</td>
</tr>
</tbody>
</table>

\(^1\)Additionally, the backoff procedure counter (BPC) is used to identify the current backoff stage.
reduce collisions induced by small contention windows. This is achieved by triggering a redraw of the backoff counter $BC$ before the station attempts a transmission.

$DC$ allows a station to enter a higher backoff stage even if it did not attempt a transmission. The mechanism to decide when this occurs works as follows. When entering backoff stage $i$, $DC$ is set at an initial $DC$ value $d_i$, where $d_i$ is given in Table I for each $i$. After having sensed the medium busy, a station decreases $DC$ by 1 (in addition to $BC$). If the medium is sensed busy and $DC = 0$, then the station jumps to the next backoff stage (or re-enters the last backoff stage, if it is already at this stage), and it re-draws $BC$ without attempting a transmission. Figure 1 shows an example of such a process.

As mentioned earlier, there are four priority classes in 1901 (see Table I). From those, CA0/CA1 priorities serve best-effort applications, and CA2/CA3 the delay-sensitive ones. The delay-sensitive class employs smaller contention windows, and the contention window is not doubled between backoff stages 1 and 2. This improves delay/jitter, but yields a higher collision probability, i.e., lower throughput, compared to CA0/CA1 class. In this paper, we focus on modeling and enhancing the throughput performance of 1901 (which is particularly relevant for best-effort applications), whereas in [8] and [9] we investigate the jitter and related short-term fairness aspects of 1901 (relevant mostly for delay-sensitive applications).

### III. RELATED WORK

Models of 802.11 can provide insights for analyzing 1901. Indeed, 802.11 can be viewed as a simplified version of 1901 in which the deferral counter never expires. We review here major studies on 802.11 and some of the few studies on 1901.

#### A. Models of IEEE 802.11

A large number of performance evaluation models have been proposed for 802.11 (e.g., [4], [5]). Among those, the model proposed by Bianchi in [4] for single contention domain networks is very popular. This work models the backoff process of 802.11 using a discrete time Markov chain. The main assumption behind this model is that the backoff processes of the stations are independent, which is known as the decoupling assumption. With this assumption, the collision probability $\gamma$ experienced by all stations is time-invariant, and can be found by solving a fixed-point equation that depends on the protocol parameters.

Kumar et al. [5] study the backoff process of 802.11 under the decoupling assumption using renewal theory. We employ a similar method for finding a fixed-point equation for 1901, which is significantly more challenging than that for 802.11 due to the increased complexity of 1901.

#### B. Models of IEEE 1901

The works analyzing the backoff process of 1901 are [8] and [10]–[12]. The authors in [10] propose a model similar to Bianchi’s model for 802.11 [4]. However, the paper does not provide the corresponding fixed-point equation for the collision probability, due to the increased complexity of the Markov chain resulting from introducing the deferral counter.

To compute the collision probability in this case, a costly system of more than a thousand non-linear equations has to be solved. Moreover, it has not been investigated whether this system of equations has a unique solution. The work in [11] discusses the validity of the decoupling assumption and simplifies the model of [10] under non-saturated assumptions. We propose a model which is strictly equivalent to the model of [10] in terms of accuracy. The key difference is that, in our case, the collision probability can be obtained by solving a single fixed-point equation. In this sense, our model can be seen as a drastic simplification of [10], and this simple form enables us to derive efficient configuration parameters for 1901. In [8], we introduce an alternative model that does not rely on the decoupling assumption and, as a result, is more accurate. However, this model is complex and does not yield insights for understanding optimality and enhancing performance.

#### C. Enhancements of IEEE 802.11

There is a large body of work introducing enhancements for the 802.11 CSMA/CA. In particular, Bianchi [4] computes the optimal contention window that achieves maximum throughput, which is a function of the number of contending stations $N$. Typically, $N$ is unknown and varies with time, hence practical implementations of such optimal configurations use some estimation techniques.

To apply Bianchi’s analysis, several attempts have been made to estimate the number of contending stations [6], [13], [14]. These methods typically rely on measuring the collision probability or the channel activity and on estimating $N$ periodically. The main disadvantage of such approaches is that they introduce more complexity at the MAC layer, challenging their practical implementation.

#### D. Enhancements of IEEE 1901

There are not many studies on enhancements of 1901. The authors in [15] propose a constant $d_i$ equal to 0 for all backoff stages $i$, which means that whenever a station senses the medium busy it doubles $CW$. This technique decreases the collision probability, but yields the most extreme case of
1901 unfairness; for small $N$, always doubling CW leads idle stations to have fewer chances to access the channel, compared to a station that just transmitted successfully (and whose CW is minimal). This yields a high variance of delay, as explained in Section II. The authors in [16] provide a mechanism that keeps $d_i$ and CW constant, but where CW depends on the number of stations $N$. As discussed before, requiring to know $N$ is impractical.

This article extends our conference publication [1] very substantially by (i) extending the proof of uniqueness to our model for a wider range of configurations, (ii) much more thorough simulation validations of our model, (iii) evaluations of our enhancement under more traffic scenarios, and (iv) validating our performance gains on Wi-Fi hardware, over a testbed of 25 nodes. Additionally, we introduce a new analytical development to give insights on the reasons why the proposed protocol performs better than 802.11 in Section V.

IV. ANALYSIS

In this section, we introduce our model for the 1901 CSMA/CA protocol. We analyze the protocol under the following assumptions (all of them widely used [1], [4], [5], [10], [13]). First, there are $N$ saturated stations in the network (i.e., stations that always have a packet ready for transmission).² Second, all stations belong to a single contention domain. Third, there is no packet loss or errors due to the physical layer and, therefore, transmission failures are due only to collisions. Fourth, the stations never discard a packet until it is successfully transmitted, hence the retry limit is infinite. Finally, we assume that all the contending stations are in the same class and use the same set of parameters.³

We now turn our attention to the modeling assumption which is referred to as the decoupling assumption (see e.g., [4], [5]). According to this assumption, the backoff process of a station is independent of the aggregate attempt process of the other $N - 1$ stations. This yields the following approximations:

1) Given a tagged station, the probability that at least one of the other stations transmits at any time slot is fixed, denoted by $\gamma$, from which (i) transmission attempts experience a fixed collision probability $\gamma$, and (ii) a station with $BC \neq 0$ senses the medium busy at any time slot with a fixed probability $\gamma$. With this, we can compute the probability that a station transmits in a randomly chosen time slot, which we denote by $\tau$, as a function of $\gamma$.

2) The assumption also implies that the transmission attempts of different stations are independent, with the transmission probability of a station given by the average attempt rate $\tau$. This allows to express $\gamma$ as a function of $\tau$, leading to a fixed-point equation.

The above assumption, which considers that stations are decoupled, has been proven to be accurate in 802.11.

However, in [8] it has been shown that in a PLC network with few stations there is some coupling between the stations for some configurations. Indeed, for such configurations, when a station is in a state with a small backoff stage, the probability that the other stations are at large backoff stage states is high, which contradicts the decoupling assumption. The simulation results of Section VI confirm that, while the model has reduced accuracy for some configurations when there are few stations in the network, accuracy is high as long as the number of stations is not too small.

Our analysis requires computing the expected number of time slots spent by a station at backoff stage $i$ (where a time slot can either be idle or contain a transmission). Let $k$ denote the value of $BC$ drawn uniformly at random in $\{0, \ldots, CW_i - 1\}$, when the station enters stage $i$. If the station is running 802.11, the station leaves the backoff stage when (and only when) it attempts a transmission; hence, the station stays in the backoff stage exactly $k + 1$ time slots, until $BC$ expires and it attempts a transmission. In contrast, in 1901 a station might leave backoff stage $i$ either because of a transmission attempt, when $BC$ expires (like in 802.11), or because it has sensed the medium busy $d_i + 1$ times, before $BC$ has expired. In the latter case, the station spends a number of slots at backoff stage $i$ equal to $j$ if it senses the medium busy for the $(d_i + 1)^{th}$ time in the $j^{th}$ slot (where $d_i + 1 \leq j \leq k$).

Let us write $bc_i$ for the expected number of time slots spent by a station at backoff stage $i$. To compute $bc_i$, we need to evaluate the probability of the events that (i) a station attempts a transmission or (ii) senses the medium busy $d_i + 1$ times within the $k$ slots (i.e., before $BC$ expires). Let $b$ be the random variable denoting the number of busy slots within the $k$ slots. Because of the decoupling assumption, $b$ follows the binomial distribution $\text{Bin}(k, \gamma)$. Now, let $x_i^j$ be the probability that a station at backoff stage $i$ jumps to the next stage $i + 1$ in $k$ or fewer time slots due to event (ii). Then,

$$x_i^j = \mathbb{P}(b > d_i) = \sum_{j=d_i+1}^{k} \binom{k}{j} \gamma^j (1 - \gamma)^{k-j}. \quad (1)$$

We can compute $bc_i$ as a function of $\gamma$ via $x_i^k$. We distinguish two cases on $k$. First, if $k > d_i$, then event (i) occurs with probability $1 - x_i^k$, and event (ii) occurs with probability $x_i^k$. For event (i), the station spends $k + 1$ slots in stage $i$. For event (ii), the station spends $j$ slots in backoff stage $i$ with probability $x_i^j - x_i^{j-1}$ for $d_i + 1 \leq j \leq k$ (observe that $x_i^j - x_i^{j-1}$ corresponds to the probability that (ii) happens exactly at slot $j$). Second, if $k \leq d_i$, then event (ii) cannot happen. Thus, the backoff counter expires, event (i) always takes place, and the station spends $k + 1$ time slots in stage $i$. By considering all the possible cases described above, $bc_i$ can be computed as

$$bc_i = \frac{1}{CW_i} \sum_{k=d_i+1}^{CW_i-1} \left[ (k + 1)(1 - x_i^k) + \sum_{j=d_i+1}^{k} j(x_i^j - x_i^{j-1}) \right] + \frac{(d_i + 1)(d_i + 2)}{2CW_i}. \quad (2)$$

²While our analysis is limited to saturation conditions, we note that it could be extended to non-saturated scenarios following a similar approach to [17].

³IEEE 1901 specifies that only stations belonging to the highest contention priority class participate in the backoff process; in practice, the highest class is decided using a simple system of busy tones, the priority resolution symbols.
We next need to compute the probability that a station at backoff stage \( i \) ends this stage by attempting a transmission, which we denote by \( t_i \), and the probability that such a backoff stage ends with a successful transmission, which we denote by \( s_i \). Similarly to (2), \( t_i \) can be computed as

\[
t_i = \frac{d_i + 1}{C W_i} + \sum_{k=d_i+1}^{CW_i-1} \frac{1}{C W_i} (1 - x_k^i).
\] (3)

From the above, \( s_i \) can be simply computed as \( s_i = (1 - \gamma) t_i \).

Building on the above expressions for \( b c_i, t_i \) and \( s_i \), we now address the computation of the average attempt rate of a station, \( \tau \). We proceed as follows. Let \( R \) be the random variable denoting the number of transmission attempts experienced by a successfully transmitted packet. Similarly, let \( X \) be the random variable denoting the total number of slots spent in backoff for a successfully transmitted packet. Then, from the renewal-reward theorem (\( R \) being the reward and \( X \) the renewal lifetimes [5]), the average attempt rate is given by

\[
\tau = \frac{\mathbb{E}[R]}{\mathbb{E}[X]}. \tag{4}
\]

We are now ready to compute \( \mathbb{E}[R] \) and \( \mathbb{E}[X] \) with the following two lemmas. The proofs are provided in Appendix.

**Lemma 1:** The expected number of slots spent in backoff per successfully transmitted packet is

\[
\mathbb{E}[X] = \sum_{i=0}^{m-2} t_i \prod_{j=0}^{i-1} (1 - s_j) + \sum_{i=0}^{m-2} (1 - s_i) \frac{b c_{m-1}}{s_{m-1}}.
\]

**Lemma 2:** The expected number of transmission attempts per successfully transmitted packet is

\[
\mathbb{E}[R] = \sum_{i=0}^{m-2} t_i \prod_{j=0}^{i-1} (1 - s_j) + \sum_{i=0}^{m-2} (1 - s_i) \frac{t_{m-1}}{s_{m-1}} = \frac{1}{1 - \gamma}.
\]

We can now derive a fixed-point equation on \( \gamma \). From the decoupling assumption, the probability \( \gamma \) that at least one other station transmits can be expressed as a function of \( \tau \) as follows:

\[
\gamma = \Gamma(\tau) = 1 - (1 - \tau)^{N-1}.
\]

In turn, \( \tau = \mathbb{E}[R]/\mathbb{E}[X] \) can be expressed as a function of \( \gamma \) by using Lemmas 5 and 5. Let us denote this function by \( G(\gamma) \). The composition of the functions \( \tau = G(\gamma) \) and \( \gamma = \Gamma(\tau) \) yields the fixed-point equation for the collision probability

\[
\gamma = G(\gamma). \tag{5}
\]

By solving the above fixed-point equation, we can determine the \( \gamma \) value that corresponds to the operating point of the system. Theorem 1 below establishes the uniqueness of the solution of (5) under the condition that the level of aggressiveness (i.e., transmission probability) is decreasing with the backoff stage \( i \). The theorem, with proof in Appendix, provides a wide range of configurations satisfying this condition. From Table 1, these configurations are compliant with the standard, except for the class CA2/CA3 at backoff stage \( i = 1 \). This suggests that it may be worth re-evaluating this configuration choice in the standard: indeed, with class CA2/CA3 stations increase their level of aggressiveness upon jumping to backoff stage 1, which contradicts the spirit behind the protocol design of decreasing aggressiveness upon an indication of congestion.

**Theorem 1:** \( G(\gamma) : [0, 1] \rightarrow [0, 1] \) has a unique fixed-point if the following condition is satisfied for \( 0 \leq i \leq m - 2 \)

\[
\begin{align*}
CW_{i+1} &\geq CW_i & \text{if } d_{i+1} = d_i, \\
CW_{i+1} &= CW_i & \text{if } d_{i+1} < d_i, \\
CW_{i+1} &\geq 2CW_i - d_i - 1 & \text{otherwise}.
\end{align*}
\]

We now explain how to obtain actual throughput figures from our model. Once we have the value for \( \gamma \) from the fixed-point equation of (5), we can obtain \( \tau \). We can then compute \( p_s \) and \( p_c \), the probability that a slot contains a successful transmission or that it is empty, respectively, from \( p_s = N \tau (1 - \tau)^{N-1} \) and \( p_c = (1 - \tau)^N \). We can further compute the probability that a slot contains a collision as \( p_c = 1 - p_c - p_s \). We now have enough information to compute the normalized throughput \( S \) of the network as

\[
S = \frac{p_s D}{p_s T_s + p_c T_c + p_c \sigma}, \tag{7}
\]

where \( D \) is the frame duration, \( T_s \) is the duration of a successful transmission, \( T_c \) is the duration of a collision, and \( \sigma \) is the time slot duration.

V. ENHANCEMENTS OF THE IEEE 1901 MAC FOR HIGH THROUGHPUT

As highlighted in Section II, 1901 has the advantage over 802.11 in that it reacts pro-actively to collisions; however, the current configuration proposed by the standard does not exploit this advantage to obtain high throughput. In this section, we leverage our 1901 model to devise efficient configurations that perform close to an optimal MAC protocol.

A. Deriving the Proposed Configuration

In the following, we propose a configuration of the 1901 MAC parameters that drives the system to the operating point that maximizes the achievable throughput of the network. According to [4], this point is achieved when the transmission probability \( \tau \) takes the following optimal value, which we denote by \( \tau_{opt} \),

\[
\tau_{opt} \approx \frac{1}{N \sqrt{\frac{2\sigma}{T_c}}}. \tag{8}
\]

From the above, we have that the collision probability at the optimal operating point, which we denote by \( \gamma_{opt} \), is given by

\[
\gamma_{opt} = 1 - (1 - \tau_{opt})^{N-1} = 1 - \left(1 - \frac{1}{N} \sqrt{\frac{2\sigma}{T_c}}\right)^{N-1} \tag{9}
\]

which, as \( N \) gets large, can be approximated by

\[
\gamma_{opt} \approx 1 - e^{-\sqrt{\frac{2\sigma}{T_c}}}. \tag{10}
\]

For the 1901 parameters (shown in Section VI), the approximation is good for \( N \geq 3 \).
Following the analysis in Section IV, \( \gamma \) can be obtained from the fixed-point equation (5), which can be rewritten as

\[
1 - (1 - \gamma)^{1/(N - 1)} = G(\gamma). \tag{11}
\]

Figure 2 illustrates the solutions of the above equation for (i) 1901 under the default configuration (‘1901 default’); (ii) 1901 under the configuration proposed here (‘1901 proposed’); and (iii) 802.11 under its default configuration (‘802.11 default’), for various \( N \) values (\( N = 2, 7, 12, \ldots, 27 \)).

To drive the 1901 network towards the optimal operation point derived above, we would like the \( \gamma \) value that solves (11) to be as close as possible to \( \gamma = \gamma_{opt} \) for all \( N \) values. To force this, we would like that \( G(\gamma) \) resembles as much as possible a step function that takes very large values for \( \gamma < \gamma_{opt} \), decreases sharply to a very small value at \( \gamma = \gamma_{opt} \), and takes a very small value from this point on. An example of such an optimal step function is illustrated in Figure 2 (‘G(\gamma) optimal’). Note that \( G(\gamma) \) is given by \( \mathbb{E}[R]/\mathbb{E}[X] \), and its shape depends on the setting of the MAC parameters. In the following, we derive the parameter configuration that drives \( G(\gamma) \) as close to the desired behavior as possible. To achieve this, we proceed as follows. \( G(\gamma) \) can be recast as

\[
G(\gamma) = \frac{1}{(1 - \gamma) \sum_{i=0}^{\infty} bc_{\min,i}(m - 1) \prod_{j=0}^{i-1} [1 - (1 - \gamma) t_j(\gamma)]}, \tag{12}
\]

where \( t_j \) is given by (3) and \( bc_{\min,i}(m - 1) = bc_i \) if \( 0 \leq i \leq m - 1 \) and \( bc_{m-1} \) otherwise.

From the above expression, it can be seen that our goal will be achieved if \( t_j(\gamma) \) follows a step function that is as close to 1 as possible for \( \gamma < \gamma_{opt} \) and as close to 0 as possible for \( \gamma > \gamma_{opt} \). With this, \( G(\gamma) \) will take the largest possible value for \( \gamma < \gamma_{opt} \) and the smallest possible value for \( \gamma > \gamma_{opt} \). The value of \( t_i \) is computed as

\[
t_i = \frac{1}{CW_i} \sum_{j=0}^{CW_i - 1} P(d_i, j, \gamma), \tag{13}
\]

where \( P(d_i, j, \gamma) \) is the probability that a station senses no more than \( d_i \) transmissions in \( j \) slots, given that the collision probability is equal to \( \gamma \).

From the analysis of Section IV, the number of transmissions that a station senses in \( j \) slots follows the binomial distribution \( \text{Bin}(j, \gamma) \); hence, its probability mass function (pmf) has a peak around the mean value, \( j \gamma \), and has a variance of \( j\gamma(1 - \gamma) \). The term \( P(d_i, j, \gamma) \) is the probability that the station senses no more than \( d_i \) transmission in \( j \) slots. Then, the key approximation that we make to derive the proposed configuration is to assume that the variance is sufficiently small such that \( P(d_i, j, \gamma) \) is equal to 0 when \( d_i \) is below the peak and equal to 1 otherwise. Note that for typical configurations we have \( \gamma_{opt} \ll 1 \), which makes the variance small and thus helps to make this approximation accurate.

With the above approximation, we have

\[
t_i \approx \frac{1}{CW_i} \sum_{j=0}^{CW_i - 1} 1_{j \gamma < d_i + 1} = \frac{d_i + 1}{\gamma CW_i}, \tag{14}
\]

which (as desired) takes a large value for small \( \gamma \) and a small value for large \( \gamma \). To force that the transition from large values (around 1) to small ones (around 0) takes place at \( \gamma = \gamma_{opt} \), we impose that \( t_i \) takes an intermediate value, i.e., \( t_i = 1/2 \), at this point, which yields

\[
t_i |_{\gamma = \gamma_{opt}} = \frac{d_i + 1}{\gamma_{opt} CW_i} = \frac{1}{2}, \tag{15}
\]

from which

\[
d_i = \frac{CW_i}{2} \gamma_{opt} - 1. \tag{16}
\]

With the above setting of \( d_i \), if the current transmission probability in the network is too high, stations jump to the next backoff stage without transmitting. To enforce that this reduces the overall transmission probability, we need to make sure that the \( CW_i \) of the next backoff stage is larger than the current one. However, to keep a sufficient level of granularity in the transmission probabilities, the increase of \( CW_i \) should not be too drastic (as otherwise a station might jump from a \( CW_i \) value that is too small to one that is too large). Following this argument, we set \( CW_{i+1} = 2CW_i \). This yields

\[
d_i = \left[ \gamma_{opt} \frac{2^i CW_{min}}{2} - 1 \right], \tag{17}
\]

where the ceiling is used to avoid negative values of \( d_i \).

From the above, we have fixed the \( d_i \) values. The remaining challenge is to configure \( CW_{min} \), and \( m \). To ensure that the curve \( 1 - (1 - \gamma)^{1/(N - 1)} \) crosses \( G(\gamma) \) at \( \gamma = \gamma_{opt} \), we need that \( t_{max} \) is sufficiently large so that even with small \( N \) the curve is crossed at this point; conversely, we also need that \( t_{min} \) is sufficiently small to cover the cases with large \( N \).

To guarantee that \( t_{max} \) is sufficiently large even for \( N = 2 \), the \( CW \) at backoff stage 0, \( CW_{min} \), needs to be as small as the optimal \( CW \) for \( N = 2 \). Accordingly, we set \( CW_{min} \) equal to the optimal \( CW \) value for \( N = 2 \), this gives \( \gamma_{opt} = \tau = 2/(CW_{min} + 1) \), from which

\[
CW_{min} = \left\lfloor \frac{2}{1 - e^{-\sqrt{\tau}}}/2 - 1 \right\rfloor. \tag{18}
\]
Finally, we need to ensure that $\tau_{\text{min}}$ is sufficiently small to provide good performance for large number of stations. To achieve this, $m$ needs to be sufficiently large. In this paper, we choose the configuration $m = 6$: with this setting for $m$, the resulting $\tau_{\text{min}}$ is sufficiently small to ensure that $\tau = 1 - (1 - \gamma)^{1/(N-1)}$ crosses $G(\gamma)$ close to $\gamma = \gamma_{\text{opt}}$ even for $N$ as large as 30.

Figure 2 shows the point of operation resulting from our configuration as well as the default ones for 1901 and 802.11, given by the intersection between the curves $1 - (1 - \gamma)^{1/(N-1)}$ and $G(\gamma)$. While the goal of our configuration is that $G(\gamma)$ decreases sharply at $\gamma = \gamma_{\text{opt}}$, we can observe that this decrease is smoothed by the randomness associated to the deferral and backoff counters. Despite this, our configuration is still much closer to $\gamma_{\text{opt}}$ (and hence to the point of optimal performance) than the default configurations.

B. 1901-Proposed Advantage Over 802.11 DCF

As we argued above, the key to optimize performance is to force that $\mathbb{E}[R]/\mathbb{E}[X]$ decreases from a large value to a small one at $\gamma = \gamma_{\text{opt}}$ as sharply as possible. In the following, we show that, thanks to the deferral counter mechanism, 1901 achieves a sharper transition, which is why it provides better performance to 802.11 across different $N$ values.

Equation (12) can be rewritten as

$$G(\gamma) = \frac{\mathbb{E}[R]}{\mathbb{E}[X]} = \frac{1}{\sum_{i=0}^{\infty} (1 - \gamma)bc_{\text{min}}(m, l_i) \prod_{j=0}^{l_i} n_j},$$

where $n_j$ is the probability that, when the station moves out of backoff stage $j$, it jumps to $j + 1$ instead of returning to backoff stage 0. From the above expression, it can be seen that $G(\gamma)$ depends on $\gamma$ through: (i) the $(1 - \gamma)bc_{\text{min}}(m, l_i)$ terms, which decrease with $\gamma$, and (ii) the $n_j$ terms, which increase with $\gamma$. From the proof of Theorem 1, it can be seen that $G(\gamma)$ strongly decreases with $\gamma$, which shows that the main dependency of $G$ on $\gamma$ comes from the $n_j$'s.

Now, we analyze the behavior of $n_j$ for both 1901 and 802.11, in order to understand the different performance of the two protocols. For 802.11 DCF, $n_j$ is given by the probability that a transmission attempt collides, i.e.,

$$n_{\text{dcf}} = \gamma, \text{ for all } j.$$

For 1901, $n_j$ is not only driven by the collision probability $\gamma$, but also by the deferral counter expiry probability $1 - t_j$, i.e.,

$$n_{\text{plc}} = 1 - (1 - \gamma)t_j(\gamma),$$

which, combined with (16), yields

$$n_{\text{plc}} = 1 - (1 - \gamma)d_j + \frac{1}{\gamma}C W_j.$$

For $G(\gamma)$ to decrease sharply with $\gamma$ at $\gamma = \gamma_{\text{opt}}$, we would like that $n_j$ increases as sharply as possible. If we look at the partial derivative of $n_j$ with respect to $\gamma$ evaluated at $\gamma = \gamma_{\text{opt}}$ for 802.11 and 1901, we obtain, respectively

$$\left.\frac{\partial n_{\text{plc}}}{\partial \gamma}\right|_{\gamma=\gamma_{\text{opt}}} = 1,$$

and

$$\left.\frac{\partial n_{\text{plc}}}{\partial \gamma}\right|_{\gamma=\gamma_{\text{opt}}} = \frac{1}{2} + \frac{1 - \gamma_{\text{opt}}}{2\gamma_{\text{opt}}^2}.$$

Given that $\gamma_{\text{opt}} \ll 1/2$, from the above we have $\partial n_{\text{plc}}/\partial \gamma \gg \partial n_{\text{dcf}}/\partial \gamma$, i.e., the probability of jumping to the next backoff stage increases much more sharply with 1901 than with 802.11, which explains the performance improvement achieved by 1901 over 802.11. This behavior is caused by the fact that while with 802.11 the parameter $\gamma$ only affects the probability of jumping to the next backoff stage through the collision probability, with 1901 $\gamma$ affects this probability not only through the collision probability but also through the probability that a station jumps to the next backoff stage upon expiring the deferral counter.

VI. PERFORMANCE EVALUATION

In this section, we evaluate the accuracy of our model and the performance of our proposed 1901 configurations via simulation as well as by means of a testbed of 802.11 devices. In [1], our simulator and model are validated experimentally in a testbed of PLC devices.

The rest of this section is structured as follows. First, we validate the accuracy of our model against simulations. Second, we extensively study the 1901 proposed configurations by (i) comparing our proposed configurations with the performance of an optimal 802.11 protocol and the default 1901, and (ii) conducting an exhaustive search in the parameter space to confirm that our configuration performs closely to the one maximizing throughput across different numbers of stations. Third, we analyze the convergence time of our 1901 enhancements and compare it against the dynamic algorithm of [6]; in contrast to this kind of adaptive algorithms, 1901 adapts the behavior based on the sensed transmissions and not on estimation techniques, which drastically reduces convergence time. Finally, we implement the 1901 CSMA/CA protocol on a testbed of WiFi hardware and show that it outperforms the default 1901 configuration in practice.

A. Validation of the 1901 Model

We now present the normalized throughput obtained by our model and via simulation. For the simulations, we assume that the same physical rate is used for all packets and take the time slot duration and timing parameters specified by the standard (see Table II). A PLC frame transmission has a duration $D$ and is preceded by two priority tone slots ($P_{RS}$) and a preamble ($P$), and followed by a response inter-frame space ($RIFS$), the ACK, and finally, the contention inter-frame space ($CIFS$). Thus, a successful transmission has a duration $T_c = 2P_{RS} + P + D + RIFS + ACK + CIFS$. In case of a collision, the stations defer their transmission for $1EIFS$ $\mu$s, where $1EIFS$ is the extended inter-frame space used by 1901; hence, a collision has a duration $T_c = 1EIFS$. Figure 3 compares the normalized throughput of 1901 obtained via simulation against our model, for (i) the default

5Note that our proposed enhancement can be used for different data-rate HomePlug specifications, since they all employ the same CSMA/CA process.
parameters for the two priority classes CA1 and CA3 (CA0 and CA2 are equivalent), and (ii) some additional configurations. We observe a good fit between analysis and simulation, with an exception for small $N$ and small $d_i$’s (CA0/CA1 class and $CW_{min}$ = 8, $d_5 = 0$, $m = 4$ configuration). The fact that the accuracy is somewhat reduced for small $N$ in these cases is due to the decoupling assumption, which fails to capture the coupling introduced by the deferral counter (see [8] for a detailed discussion on this issue).

B. Proposed Enhancement for the IEEE 1901 MAC

We now evaluate the enhancements proposed in Section V. Given the parameters of Table II and the expressions (17) and (18), the parameters of the enhanced 1901 are $CW_{min} = 12$ and $d_i = \{0, 1, 3, 6, 13, 27\}$ for each backoff stage, with $m = 6$ (as mentioned earlier, our enhancements consist simply in modifying the parameters of the protocol).

1) Proposed vs Default 1901: Figure 4 compares the performance of (i) 1901 using default CA1 configuration, (ii) 1901 using our proposed enhanced configurations mentioned above, and (iii) “optimal” 802.11 for a varying number of stations – for the “optimal” 802.11, $CW_{min}$ is computed from (8) [4], [6]. It appears that 1901 with our proposed configuration performs similarly – or better than – the optimally configured 802.11, which requires knowing the number of stations $N$. Furthermore, the proposed configurations drastically boost the efficiency of 1901.

In Figure 4, we also show similar results, but using $m = 4$ instead of $m = 6$ for the number of backoff stages. Here too, our proposed configuration yields substantial improvements. These improvements can offer tens of Mbps of throughput gain, given the data rate (up to 1Gbps) of HomePlug AV2.

2) Exhaustive Search of Optimal Configuration: To further assess the performance of our proposed configuration, we run an exhaustive search in the parameter space of the $CW_{min}$ and $d_i$’s; in particular, we take $m = 6$, $CW_i = 2^i CW_{min}$, and $d_i = \lceil f^i - 1 \rceil$, $i \in \{0, 5\}$, where $f \in \mathbb{R}$, and run the exhaustive search for $2 \leq CW_{min} \leq 32$ and $1 \leq f \leq 5$.

Our aim is to maximize throughput. However, since different numbers of stations have a maximum throughput at different configurations, we need a criterion to find a good trade-off between the performance for different numbers of stations. To this end, we define function $\sum_{n=1}^{N} \log(S_n)$, where $S_n$ is the normalized throughput of a scenario with $n$ stations, and select the configuration that maximizes this function (computed with our model, for $N = 30$).

To validate our proposal, we compare it against the configuration resulting from the above criterion, obtained by performing an exhaustive search. The values returned by this exhaustive search are $CW_{min} = 14$ and $f = 1.5$. Figure 5 presents the normalized throughput of the default configuration, the proposed one and the exhaustive search. We observe that our proposed configuration performs very close to the one returned by our exhaustive search algorithm; while the configuration resulting from the exhaustive search slightly outperforms ours, it should be noted that it is also less fair and it increases jitter (e.g., by 25% for $N = 2$) as compared to our proposal.6

C. Evaluation Under Dynamic Traffic

The evaluation of the proposed configuration in the previous sections has been limited to static scenarios with a fixed

\[ \text{Table II: Simulation Parameters} \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Duration (\mu s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slot $\sigma$, Priority slot $PRS$</td>
<td>35.84</td>
</tr>
<tr>
<td>$CIFS$</td>
<td>100.00</td>
</tr>
<tr>
<td>$RIFS$</td>
<td>140.00</td>
</tr>
<tr>
<td>Preamble, $ACK$</td>
<td>110.48</td>
</tr>
<tr>
<td>Frame duration $D$</td>
<td>2500.00</td>
</tr>
<tr>
<td>$EIFS$</td>
<td>2920.64</td>
</tr>
</tbody>
</table>

\[ \text{Fig. 3: Normalized throughput obtained by simulation and with our model, for the default configurations of 1901 (top) and for various configurations (bottom). Lines represent simulations and points the analytical results.} \]

\[ \text{Fig. 4: Simulations of 1901, enhanced 1901 proposed here, and optimal 802.11 for } m = 6 \text{ (left) and } m = 4 \text{ (right).} \]
Fig. 5. Performance comparison of the proposed 1901 configuration, the configuration that maximizes throughput obtained from an exhaustive search, and the default 1901 configuration.

Fig. 6. Performance comparison of enhanced 1901 proposed here and of the DAC 802.11 algorithm under dynamic traffic. At 16 s (shown by the vertical line), the number of saturated stations changes drastically from 10 to 2. The dashed rectangle shows the convergence time of the DAC algorithm, which is in order of a few seconds after the change in traffic demand.

number of stations. However, in reality the number of active stations is not fixed, but it varies with time. One of the important advantages of the proposed enhancement as compared to other optimal adaptive approaches is that 1901 adapts the behavior with a very fine granularity: it updates $CW_{\text{min}}$ after sensing transmissions, which yields fast adaptation to varying traffic. In contrast, adaptive algorithms for 802.11 require to update their estimation periodically after having enough samples of collided/transmitted frames, which can yield long convergence times under dynamic traffic.

In the following, we evaluate the convergence time of our approach and compare it against one representative adaptive algorithm for 802.11: the Distributed Adaptive Control (DAC) algorithm of [6], which estimates the collision probability and drives the system to its optimal value by adjusting $CW_{\text{min}}$ periodically (every 100 ms). For this evaluation, we run simulations in which we drastically change the number of stations from 10 to 2. Figure 6 shows the average $CW$ (averaged over all stations and over 200 ms intervals) and the instantaneous throughput (averaged over 200 ms intervals) obtained from these simulations. Results confirm that existing 802.11 adaptive algorithms fail to react quickly to changing scenarios, harming the resulting performance over transients, and in contrast, our approach reacts much faster.

D. Evaluation Under Non-Saturated Traffic

So far, our evaluation has focused on saturated scenarios in which all stations always have a packet ready for transmission. In order to evaluate the performance of the proposed enhancement under non-saturated conditions, in the following we consider bursty ON/OFF traffic for all stations, where a station generates 10 frames during an ON period and OFF durations follow (i) an exponential distribution, or (ii) a Pareto with finite mean and infinite variance (i.e., a long-tailed distribution). Figure 7 shows the normalized throughput as a function of the offered load in a network with $N = 10$ stations. We conclude from the results that (i) as long as the offered load falls below the saturation throughput, our enhancement succeeds in delivering 100% of the offered load, and (ii) otherwise, the throughput provided corresponds to the saturation throughput. These results confirm that our enhancement works well for these scenarios. In [1], we have provided additional results for mixed scenarios where some stations are saturated and the others are non-saturated.

E. Experimental Validation of Proposed Enhancement

To confirm the improvements shown via simulation in the previous sections, we implement the 1901 protocol with the proposed configuration in WiFi hardware. The reason for choosing this platform for our experimental validation is that to implement our enhanced configuration of 1901 we need full access to the firmware, which commercial 1901 devices do not provide. Therefore, we employ 802.11 Broadcom wireless cards and substitute the default proprietary firmware with OpenFWWF [18] that has already been used to extend and modify the 802.11 default behavior [19], [20].

The assembly code of the firmware has been modified to follow the 1901 protocol. To this end, we store the different contention window ($CW_i$) and deferral counter ($d_i$) values into the shared memory and tweak the $\text{rx}\_\text{plcp}$ and $\text{tx}\_\text{plcp}$
update_contention_params modules to implement the backoff procedure defined in 1901 after a data transmission or when an ongoing data transmission is sensed in the channel. 

With the above implementation, we deploy our testbed of Alix 2d2 devices from PC Engines, each with a Broadcom BCM94318MPG 802.11b/g MiniPCI wireless card and with the Ubuntu 10.04 Linux (kernel 2.6.36) distribution installed. The testbed is located under a raised floor of a laboratory and it comprises 25 wireless stations, and one desktop machine that acts as an access point (AP) (see Figure 8). We carefully run our experiments when the channel activity is very low to avoid any external source of interference. To generate traffic and evaluate performance, we use mgen [21] configured to send uplink saturated UDP traffic.

To emulate the PLC PHY/MAC timing parameters, we set the slot duration and fix the data-rate of WiFi to a small value, such that the slot duration is 20 μs and the duration of a successful transmission or collision is 2720 μs, which are very close to the ones of the IEEE 1901 standard (these and the rest of the transmission parameters are given in Table III). The parameters of the enhanced 1901 in this case are CWmin = 16 and di = {0, 1, 3, 7, 14, 29} for each backoff stage, with m = 6. For each experiment, we perform 10 runs. For all results, 95% confidence intervals are below 1%.

The results of this experiment, given in Figure 9, are very similar qualitatively to the simulation results shown earlier and confirm that (i) the proposed configuration maintains very good throughput performance as N increases, (ii) it outperforms very substantially the default 1901 configuration, and (iii) it performs very closely to the optimal 802.11 configuration.

VII. Conclusion

The MAC layer of IEEE 1901 can react to contention with a fine granularity and without involving collisions, which offers the potential of high gains in terms of throughput as compared to 802.11. Unfortunately, with the default parameter setting of 1901, the protocol operates far from optimality and does not fully exploit its potential. One possible reason for this is the lack of a simple model of 1901 that can be leveraged to find an appropriate setting of its parameters. Indeed, despite the commercial success of 1901 and its wide adoption in home networks, this protocol has remained largely unexplored by the research community.

One of the main challenges to model 1901 is the complexity of the protocol, which has a very large state-space. To reduce the state-space and come up with a simple model, we make the assumption that the backoff processes of the stations are independent. Building on the resulting model, we derive a procedure to steer the network towards its optimal point of operation. With this, we obtain a protocol that provides performance close to optimal independently of the number of stations while reacting quickly to changes, which is a result that has been long pursued by the research community.

Our proposal only requires modifying existing parameters, and does not change the CSMA/CA 1901 algorithm itself. Therefore, it can be easily incorporated into practical deployments. We implement the 1901 protocol on a testbed of WiFi hardware. Our simulations and testbed measurements confirm the drastic performance improvements of our proposal.

APPENDIX

Lemma 3: Let Bi be the expected number of backoff slots between two transmissions attempts of a station that always remains at backoff stage i. Then, Bi is given by bi/τi − 1, and Bi is an increasing function of γ for any i.

Proof: By its definition, Bi is given recursively by

\[
B_i = \frac{d_i(d_i + 1)}{2CW_i} + \frac{CW_{i-1}}{CW_i} \sum_{j=d_i+1}^{CW_i} \left( j(1-x_j^i) + \sum_{k=d_i+1}^{j-1} (k + B_i)(x_k^i - x_{k-1}^i) \right) / CW_i.
\]

Now, solving (19) over Bi, gives Bi = bi/τi − 1, with bi and τi given by (2) and (3).

To prove the second part of the lemma, we proceed as follows. (i) First, we compute d Bi/dγ. (ii) Second, we show that this derivative is positive at γ = 1. (iii) Third, we show that if the derivative is negative at some 0 < γ* < 1, it will also be negative at any γ > γ*. The proof then follows by contradiction: if the derivative was negative at some γ*, it would also be negative at γ = 1, which would contradict (ii).

(i) After rearranging terms, (19) can be rewritten as

\[
B_i = \frac{CW_i - 1}{2} + \frac{1}{CW_i} \sum_{j=d_i+1}^{CW_{i-1}} \left( \frac{B_i x_j^i}{j} - \sum_{k=d_i+1}^{j-1} x_k^i \right).
\]

TABLE III

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>R_min</td>
<td>Data rate</td>
<td>5.5 Mbps</td>
</tr>
<tr>
<td>R_max</td>
<td>Control rate</td>
<td>2 Mbps</td>
</tr>
<tr>
<td>L</td>
<td>Packet length</td>
<td>1460 B</td>
</tr>
<tr>
<td>σ</td>
<td>Slot duration</td>
<td>20 μs</td>
</tr>
<tr>
<td>T_preamble</td>
<td>Preamble duration</td>
<td>192 μs</td>
</tr>
<tr>
<td>T_c</td>
<td>Collision duration</td>
<td>2720 μs</td>
</tr>
<tr>
<td>-</td>
<td>Band</td>
<td>2.4 GHz</td>
</tr>
<tr>
<td>-</td>
<td>Transmission power</td>
<td>20dBm</td>
</tr>
</tbody>
</table>
The derivative of $B_i$ can be computed as
\[
\frac{dB_i}{d\gamma} = \sum_{k=d_i+1}^{CW_i-1} \frac{x_{i,k}^{j}}{CW_i} \frac{dx_{i,k}}{d\gamma}.
\]

The partial derivative $\partial B_i/\partial x_{i,k}^{j}$ can be computed from (20) as
\[
\frac{\partial B_i}{\partial x_{i,k}^{j}} = B_i - (CW_i - 1 - k) \frac{x_{i,k}^{j}}{CW_i} + \frac{\partial B_i}{\partial x_{i,k}^{j}} \sum_{j=d_i+1}^{CW_i-1} \frac{x_{i,k}^{j}}{CW_i},
\]
which yields
\[
\frac{dB_i}{d\gamma} = \sum_{k=d_i+1}^{CW_i-1} (B_i - (CW_i - 1 - k)) \frac{dx_{i,k}^{j}}{d\gamma} \frac{dx_{i,k}^{j}}{d\gamma}. (21)
\]

To compute $dx_{i,k}^{j}/d\gamma$, we observe that $x_{i,k}^{j}$ is the complementary cumulative distribution function of a binomial distribution. By taking its partial derivative, we obtain
\[
\frac{dx_{i,k}^{j}}{d\gamma} = \frac{k^j}{(k-d_i+1)!} \gamma^{d_i} (1 - \gamma)^{k-d_i-1}. (22)
\]

(iii) Next, we show that $dB_i/d\gamma > 0$ at $\gamma = 1$. At $\gamma = 1$, we have $B_i = CW_i - d_i/2 - 1$ from (19). Substituting in (21) yields $dB_i/d\gamma = d_i/2 + 1$, i.e., $dB_i/d\gamma > 0$.

(iii) Let us now assume that $dB_i/d\gamma < 0$ for some $\gamma^* < 1$. Let $l = \lceil CW_i - 1 - B_i(\gamma^*) \rceil$. Given (21), we can express $dB_i/d\gamma$ as the product of two terms, $dB_i/d\gamma = f_1(\gamma) f_2(\gamma)$, where
\[
f_1(\gamma) = \frac{dx_{i,k}^{j}}{d\gamma} \frac{CW_i - \sum_{j=d_i+1}^{CW_i-1} x_{i,k}^{j}}{CW_i},
\]
\[
f_2(\gamma) = \frac{\sum_{k=d_i+1}^{CW_i-1} (B_i - (CW_i - 1 - k)) \frac{dx_{i,k}^{j}}{d\gamma} \frac{dx_{i,k}^{j}}{d\gamma}}{CW_i}.
\]

We have $f_1(\gamma) > 0 \forall \gamma$, which implies $dB_i/d\gamma < 0$ if and only if $f_2(\gamma) < 0$. Also, we have
\[
\frac{df_2(\gamma)}{d\gamma} = \sum_{k=d_i+1}^{CW_i-1} \frac{dB_i}{d\gamma} \frac{dx_{i,k}^{j}}{d\gamma} \frac{dx_{i,k}^{j}}{d\gamma} + \sum_{k=d_i+1}^{CW_i-1} (B_i - (CW_i - 1 - k)) \frac{dx_{i,k}^{j}}{d\gamma} \frac{dx_{i,k}^{j}}{d\gamma} + \sum_{k=d_i+1}^{CW_i-1} (B_i - (CW_i - 1 - k)) \frac{dx_{i,k}^{j}}{d\gamma} \frac{dx_{i,k}^{j}}{d\gamma},
\]
\[
\frac{d}{d\gamma} \left( \frac{dx_{i,k}^{j}}{d\gamma} \frac{dx_{i,k}^{j}}{d\gamma} \right) = \frac{k!(l-d_i-1)!}{l!(k-d_i-1)!} (1 - \gamma)^{k-l-1},
\]
which is positive for $k < l$ and negative for $k > l$. From the above equations, it follows that as long as $CW_i - 1 - (l-1) > B_i(\gamma) > CW_i - 1 - l$ and $dB_i/d\gamma < 0$, we have $df_2(\gamma) < 0$.

Building on the above, next we show that $B_i(\gamma)$ decreases for $\gamma \in \{\gamma^*, \gamma^'\}$, where $\gamma^l$ is the value for which $B_i(\gamma^l) - (CW_i - 1 - 0) = 0$. At $\gamma = \gamma^*$, we have $f_2(\gamma^*) < 0$, $dB_i/d\gamma < 0$ and $df_2/d\gamma < 0$. Let us assume that, before $B_i(\gamma)$ decreases down to $CW_i - 1 - l$, there is some $\gamma^* > \gamma^l$ for which $dB_i/d\gamma > 0$. This implies that for some $\gamma^* \in (\gamma^*, \gamma^l)$, $f_2(\gamma)$ has to stop decreasing, i.e., $df_2(\gamma)/d\gamma = 0$. Since $f_2(\gamma)$ decreases in $[\gamma^*, \gamma^l]$, we have $f_2(\gamma) < 0$ for $\gamma \in [\gamma^*, \gamma^l]$. Thus, $B_i(\gamma)$ decreases in $[\gamma^*, \gamma^l]$. As $CW_i - 1 - (l-1) > B_i(\gamma^*) > CW_i - 1 - l$ and (by assumption) $B_i(\gamma)$ does not reach $CW_i - 1 - l$, we also have $CW_i - 1 - (l-1) > B_i(\gamma^*) > CW_i - 1 - l$, which contradicts $df_2(\gamma)/d\gamma = 0$. Hence, our assumption does not hold, and $dB_i/d\gamma < 0$ until $B_i$ reaches $CW_i - 1 - l$, i.e., $dB_i/d\gamma < 0$ for $\gamma \in [\gamma^*, \gamma^l]$. Following the same rationale for $\gamma \in [\gamma^*, \gamma^l+1]$, we can prove that $dB_i/d\gamma < 0$ for $\gamma \in [\gamma^*, \gamma^l+1]$. We can repeat this recursively to show that $dB_i/d\gamma < 0$ for $\gamma \in [\gamma^1, \gamma^2, \gamma^3]$ until reaching $\gamma = 1$, which yields a contradiction because $dB_i/d\gamma > 0$ at $\gamma = 1$ from step (ii) above.

**Corollary 1:** $B_i+1 > B_i$, if $CW_i+1 \geq 2CW_i - d_i - 1$.

**Proof:** By Lemma 3, the minimum value of $B_i+1$ is $B_i+1_{\text{min}} = (CW_i+1 - 1)/2$ at $\gamma = 0$, and the maximum value of $B_i$ is $B_i_{\text{max}} = CW_i - d_i/2 - 1$ at $\gamma = 1$. Setting $CW_i+1 \geq 2CW_i - d_i - 1$, yields $B_i+1_{\text{min}} \geq B_i_{\text{max}}$, hence $B_i+1 > B_i$ for all $\gamma \in [0, 1]$.

**Corollary 2:** $B_i+1 \geq B_i$, if $CW_i+1 = CW_i$ and $d_i+1 = d_i+1$.

**Proof:** By the proof of Lemma 3, the equality holds for $\gamma = 0$, because $B_i(0) = (CW_i+1)/2$. We now show that for $\gamma \in (0, 1]$, we have $B_i+1 > B_i$.

If we prove $B_i+1 > B_i$ when $CW_i+1 = CW_i$, $d_i+1 = d_i+1$, then the corollary follows by induction. Thus, we now show $B_i+1 > B_i$ for this case, and we proceed as follows.

Given (20), the difference $B_i+1 - B_i$ can be computed as
\[
B_i+1 - B_i = \sum_{j=d_i+1}^{CW_i-1} \left( B_{j+1}x_{i,j+1}^{k+1} - \sum_{k=d_i+1}^{j} x_{i,k}^{j+1} \right) \frac{B_{j+1}}{CW_i} - \sum_{j=d_i+2}^{CW_i-1} \left( B_{j}x_{i,j}^{k} - \sum_{k=d_i+2}^{j} x_{i,k}^{j} \right) \frac{B_{j}}{CW_i}. (23)
\]

We have $x_{i,j}^{k+1} = x_{i,j}^{k} + \gamma^{j+1} (1 - \gamma)^{l+1}$. Let $\delta_j = \gamma^{j+1} (1 - \gamma)^{l+1}$. Then, we have $\delta_j = x_{i,j+1}^{k} - x_{i,k}^{j}$. By rearranging the terms and solving over the difference $B_i+1 - B_i$ in (23), and by using the definition of $\delta_j$, we have
\[
B_i+1 - B_i = \sum_{j=d_i+1}^{CW_i-1} \left( B_{j+1} - B_{j} \right) \frac{\gamma^{j+1} x_{i,j+1}^{k+1}}{CW_i} - \sum_{j=d_i+1}^{CW_i-1} \frac{x_{i,j}^{k}}{CW_i} \delta_j. (24)
\]

By using (22), we now observe that
\[
\delta_j = \frac{dx_{i,j+1}^{k+1}}{d\gamma} \frac{\gamma}{d+1}.
\]

Substituting this in (24), yields
\[
B_i+1 - B_i = \frac{\sum_{j=d_i+1}^{CW_i-1} \left( B_{j+1} - B_{j} \right) \frac{\gamma^{j+1} x_{i,j+1}^{k+1}}{CW_i} - \sum_{j=d_i+1}^{CW_i-1} \frac{x_{i,j}^{k}}{CW_i} \delta_j}{\frac{\gamma}{d+1} \frac{dx_{i,j+1}^{k+1}}{d\gamma}} > 0,
\]
where the inequality holds by Lemma 3 and for all $\gamma \in (0, 1]$. 

Corollary 3: If \( B_{i+1} \geq B_i \), if \( CW_i \geq CW_{i+1} \) and \( d_{i+1} = d_i \).

Proof: If \( B_{i+1} > B_i \) holds for \( CW_i = CW_{i+1} \), by using induction it is easy to see that it holds for any \( CW_i > CW_{i+1} \). Thus, we prove the corollary for \( CW_i = CW_{i+1} \).

Because \( d_i = d_{i+1} \) and by using (1), we have \( x_k^i = x_k^{i+1} \) for all \( d_i + 1 \leq k \leq CW_i - 1 \). Given this, we have for the difference \( \Delta B = B_{i+1} - B_i \):

\[
\Delta B = \frac{(CWi - \sum_{j=d_i+1}^{CW_i-1} x_j^i)^2}{(CWi + 1 - \sum_{j=d_i+1}^{CW_i-1} x_j^i)(CWi - \sum_{j=d_i+1}^{CW_i-1} x_j^i)}
- \frac{(CWi + 1 - \sum_{j=d_i+1}^{CW_i-1} x_j^i)(CWi - \sum_{j=d_i+1}^{CW_i-1} x_j^i)}{(CWi + 1 - \sum_{j=d_i+1}^{CW_i-1} x_j^i)(CWi - \sum_{j=d_i+1}^{CW_i-1} x_j^i)}
- \frac{(1 - x_j^i) \sum_{j=d_i+1}^{CW_i-1} x_j^i}{(CWi + 1 - \sum_{j=d_i+1}^{CW_i-1} x_j^i)(CWi - \sum_{j=d_i+1}^{CW_i-1} x_j^i)}
+ \frac{(CWi + 1 - \sum_{j=d_i+1}^{CW_i-1} x_j^i)(CWi - \sum_{j=d_i+1}^{CW_i-1} x_j^i)}{(CWi + 1 - \sum_{j=d_i+1}^{CW_i-1} x_j^i)(CWi - \sum_{j=d_i+1}^{CW_i-1} x_j^i)}
\]

By the definition of \( x_j^i \) in (1), we have \( x_j^{CW_i} \geq x_j^i \), \( d_i + 1 \leq k \leq CW_i - 1 \), with equality at \( \gamma = 0 \). This yields \( \sum_{j=d_i+1}^{CW_i-1} x_j^i \leq (CWi - d_i - 1)x_j^{CW_i} \leq CW_i x_j^{CW_i} \).

We have:

\[
\Delta B \geq \frac{CWi(1 - x_j^{CW_i})(CWi - \sum_{j=d_i+1}^{CW_i-1} x_j^i)}{(CWi + 1 - \sum_{j=d_i+1}^{CW_i-1} x_j^i)(CWi - \sum_{j=d_i+1}^{CW_i-1} x_j^i)}
- \frac{(CWi + 1 - \sum_{j=d_i+1}^{CW_i-1} x_j^i)(CWi - \sum_{j=d_i+1}^{CW_i-1} x_j^i)}{(CWi + 1 - \sum_{j=d_i+1}^{CW_i-1} x_j^i)(CWi - \sum_{j=d_i+1}^{CW_i-1} x_j^i)}
- \frac{(1 - x_j^{CW_i})(CWi - \sum_{j=d_i+1}^{CW_i-1} x_j^i)}{(CWi + 1 - \sum_{j=d_i+1}^{CW_i-1} x_j^i)(CWi - \sum_{j=d_i+1}^{CW_i-1} x_j^i)}
+ \frac{(CWi + 1 - \sum_{j=d_i+1}^{CW_i-1} x_j^i)(CWi - \sum_{j=d_i+1}^{CW_i-1} x_j^i)}{(CWi + 1 - \sum_{j=d_i+1}^{CW_i-1} x_j^i)(CWi - \sum_{j=d_i+1}^{CW_i-1} x_j^i)}
\]

where the last inequality holds because \( x_j^i \leq 1 \), for all \( i, j \).

We now present the proof of Lemmas 5 and 5, and Theorem 1, introduced in Section IV.

Proof of Lemma 5: Let \( X \) be the random variable denoting the number of slots that a station that starts in stage \( i \) spends in backoff before transmitting its current packet successfully. With this notation, it holds \( \mathbb{E}[X] = \mathbb{E}[X_0] \). Let \( c_i \) and \( j_i \) denote the probabilities that a station at stage \( i \) ends this stage due to a collision, or due to sensing the medium \( d_i + 1 \) times, respectively. Note that \( s_i + c_i + j_i = 1 \) for all \( i \). Additionally, let \( bc_{x_i}, bc_{c_i}, \) and \( bc_{j_i} \) be the expected number of backoff slots that a station spends in backoff stage \( i \), given that the station ends up retransmitting its packet successfully transmitted, due to a collision, or due to sensing the medium busy, respectively. From the law of total probability, we have:

\[
\mathbb{E}[X] = \mathbb{E}[X_0] = bc_0 + (1 - s_0)\mathbb{E}[X_1].
\]

Repeating the above reasoning recursively for \( \mathbb{E}[X_1], \mathbb{E}[X_2], \ldots, \mathbb{E}[X_{m-1}] \), we have:

\[
\mathbb{E}[X_0] = bc_0 + (1 - s_0) \times (bc_1 + (1 - s_1)(bc_2 + \ldots (1 - s_{m-2})\mathbb{E}[X_{m-1}])).
\]

Now, by applying the same reasoning as in (25), we have:

\[
\mathbb{E}[X_{m-1}] = bc_{m-1} + (1 - s_{m-1})\mathbb{E}[X_{m-1}].
\]

Solving for \( \mathbb{E}[X_{m-1}] \), we obtain:

\[
\mathbb{E}[X_{m-1}] = bc_{m-1}/s_{m-1}.
\]

Plugging this expression into (26) concludes the proof.

Proof of Lemma 5: Similar to the proof of Lemma 1.

Proof of Theorem 1: By Brouwer’s fixed-point theorem, since \( \Gamma(G(\gamma)) \) is a continuous function, there exists a fixed-point in \([0, 1]\). Furthermore, if \( \Gamma(G(\gamma)) \) is monotone, this fixed-point is unique. As \( \Gamma(\gamma) \) is non-decreasing in \( \gamma \), it is thus sufficient to show that \( G(\gamma) \) is monotone in \( \gamma \).

Let \( Q(\gamma) = (1 - \gamma)\mathbb{E}[X] \). Then, we have \( G(\gamma) = 1/Q(\gamma) \).

Now, \( G(\gamma) \) is non-increasing in \( \gamma \) if and only if \( Q(\gamma) \) is non-decreasing in \( \gamma \), which we show in the following.

We now use Lemma 3 to express \( Q(\gamma) \) as a function of \( B_i \). Replacing \( bc_i \) with \( t_i(B_i + 1) \) in the expression for \( \mathbb{E}[X] \) and using \( s_i = (1 - \gamma)t_i \), \( Q(\gamma) \) can be rewritten as

\[
Q(\gamma) = \sum_{i=0}^{m-2} s_i (B_i + 1) \prod_{j=0}^{i-1} (1 - s_j) + \prod_{i=0}^{m-2} (1 - s_i)(B_{m-1} + 1).
\]

The derivative of \( Q(\gamma) \) with respect to \( \gamma \) is given by:

\[
\frac{dQ}{d\gamma} = \sum_{i=0}^{m-2} \frac{dB_i}{d\gamma} \prod_{j=0}^{i-1} (1 - s_j) + \prod_{i=0}^{m-2} (1 - s_i) \frac{dB_{m-1}}{d\gamma}
- \sum_{i=0}^{m-2} \sum_{j=i+1}^{m-2} \left( (B_j + 1)s_j \frac{\prod_{k=0}^{j-1} (1 - s_k)}{1 - s_i} \right)
- \prod_{i=0}^{m-2} (1 - s_j) + \prod_{j=0}^{m-2} \frac{(1 - s_j)}{1 - s_i}(B_{m-1} + 1).
\]

From Lemma 3, we have that \( dB_i/d\gamma > 0 \). Thus, the first two terms in (27) are positive and it follows that

\[
\frac{dQ}{d\gamma} = - \sum_{i=0}^{m-2} \frac{dB_i}{d\gamma}
\]

\[
\times \left[ \frac{m-2}{\prod_{j=0}^{m-2} (1 - s_j)}(B_{m-1} + 1) \right].
\]

In Lemma 3 we show that \( x_k^i \) is increasing with \( \gamma \) (see (22)). Thus, \( s_i \) is decreasing with \( \gamma \), since, from (3), it holds

\[
\frac{ds_i}{d\gamma} = -t_i - (1 - \gamma) \frac{\sum_{k=d_i+1}^{CW_i-1} d_k^i / d\gamma}{CWi} < 0.
\]

From Corollaries 1–3 and Condition (6) of Theorem 1, \( (B_i(\gamma)) \) is non-decreasing with \( i \). Combining these two properties (i.e., \( ds_i / d\gamma < 0 \), \( B_i+1 \geq B_i \)) for \( i = m - 2 \) with (28),
we have
\[
\frac{dQ}{d\gamma} \geq - \sum_{i=0}^{m-3} \frac{d s_i}{d\gamma} \frac{1}{1-s_i} \times \left[ \left( B_i + 1 \right) \sum_{j=i+1}^{m-2} \left( s_j \prod_{k=0}^{j-1} \left( 1-s_k \right) \right) - (B_i + 1) \right]
\times \left[ \prod_{j=0}^{m-2} \left( 1-s_j \right) + \prod_{j=0}^{m-2} \left( 1-s_j \right) (B_{m-1} + 1) \right].
\]

Using \( d s_i/ d\gamma < 0 \), \( B_{i+1} \geq B_i \) in the above inequality and rearranging the factors in products involving the \( s_i \)’s, we have
\[
\frac{dQ}{d\gamma} \geq - \sum_{i=0}^{m-3} \frac{d s_i}{d\gamma} \frac{1}{1-s_i} \times \left[ \left( B_i + 1 \right) \sum_{j=i+1}^{m-2} \left( s_j \prod_{k=0}^{j-1} \left( 1-s_k \right) \right) - (B_i + 1) \right]
\times \left[ \prod_{j=0}^{m-2} \left( 1-s_j \right) + \prod_{j=0}^{m-2} \left( 1-s_j \right) (B_{m-1} + 1) \right]
= - \sum_{i=0}^{m-3} \frac{d s_i}{d\gamma} \prod_{j=i+1}^{m-2} \left( 1-s_j \right) \left( B_{m-1} - B_i \right) \geq 0,
\]
with equality at \( \gamma = 0 \). This completes the proof.

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## REFERENCES


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