A Channel Assignment and Routing Algorithm for Energy Harvesting Multi-Radio Wireless Mesh Networks

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Abstract—Wireless mesh networks are being deployed all around the world both to provide ubiquitous connection to the Internet and to carry data generated by several services (video surveillance, smart grids, earthquake early warning systems, etc.). In those cases where fixed power connections are not available, mesh nodes operate by harvesting ambient energy (e.g., solar or wind power) and hence they can count on a limited and time-varying amount of power to accomplish their functions. Since we consider mesh nodes equipped with multiple radios, power savings and network performance can be maximized by properly routing flows, assigning channels to radios and identifying nodes/radios that can be turned off. Thus, the problem we address is a joint channel assignment and routing problem with additional constraints on the node power consumption, which is NP-complete. In this paper we propose a heuristic, named MP-CARA (Minimum Power Channel Assignment and Routing Algorithm), which is guaranteed to return a local optimum for this problem. Based on a theoretical analysis that we present in the paper, which gives an upper bound on the outage probability as a function of the constraint on power consumption, we can guarantee that the probability that a node runs out of power with MP-CARA falls below a desired threshold. The performance of MP-CARA is assessed by means of an extensive simulation study aiming to compare the solutions returned by MP-CARA to those found by other heuristics proposed in the literature.

Index Terms—multi-radio wireless mesh networks, energy efficiency, channel assignment

I. INTRODUCTION

Wireless Mesh Networks (WMNs) enable to wirelessly cover large areas with a low deployment and maintenance cost. Due to such a feature, WMNs are suited both to provide ubiquitous connection to the Internet and to carry data generated by several services (video surveillance, smart grids, earthquake early warning systems, etc.). In many of these cases, mesh nodes need to be deployed in areas where fixed power connections are not available and hence they operate by harvesting energy from the environment (e.g., solar or wind power). Mesh nodes are continuously supplied with an amount of power that is, however, limited, time-varying and dependent on how the node is provisioned. For instance, in case of a solar powered mesh node, the amount of harvested power depends on the solar panel size and the solar insolation. To account for the random nature of the harvested power, nodes are equipped with a battery of fixed capacity. If the harvested power exceeds the power consumed by the node in a certain time period, the power in excess is stored in the battery. Otherwise, the battery provides the supplementary energy required by the node to operate. However, if the power consumption persistently exceeds the harvested power, the energy stored in the battery depletes and the node runs out of power. It is of paramount importance to avoid power outages, due to the dramatic deterioration of the network performance they can cause. In order to limit the probability of power outages, it is necessary to constrain the amount of power that a node consumes. A first problem that arises is how to determine the maximum power consumption allowed to a node that ensures that the probability of a power outage is below a certain value. One of the key contributions of this paper is a theoretical analysis that allows to address such a problem. In particular, by modeling the energy levels of the battery as the state space of a Markov chain, we derive an upper bound to the outage probability as a function of the maximum allowed power consumption, the battery capacity and the probability distribution of the harvested power.

Once determined the maximum amount of power that each node can consume in order to guarantee a desired outage probability, another problem that arises is how to ensure that each node actually does not consume more than such amount of power. To this end, a number of techniques can be adopted. One of these techniques is to properly route traffic flows. For instance, flows can be routed away from nodes having more stringent power constraints in order to reduce their load and hence their power consumption. In the context of multi-radio WMNs, which are gaining popularity due to the throughput increase they enable, another means to save power is to put individual radios in a sleep state (turned off henceforth) if traffic flows can be routed in such a way to avoid using those radios. Since another primary goal is to optimize the network performance, which is highly impacted by the way channels are assigned to radios, we address the problem how to route traffic flows and how to assign channels to radios in such a way to maximize the network performance while satisfying the constraints on the maximum power consumption allowed to the nodes. In order to be conservative and overcome possible time periods when the consumed power persistently exceeds the harvested power, it is desirable that the whole network operates as efficiently as possible from an energy consumption standpoint. Hence, among all the network configurations achieving the optimal network performance while satisfying...
the constraints on the node power consumption, we aim to identify the one minimizing the total power consumption of the whole network. To the best of our knowledge, such a problem has not been investigated in the literature before.

The problem we address is a joint channel assignment and routing problem, which unfortunately is NP-complete [1]. Another key contribution of this paper is MP-CARA (Minimum Power Channel Assignment and Routing Algorithm), a heuristic to solve the addressed problem. MP-CARA takes as input the set of traffic demands and the current channel assignment, and computes how to route the traffic flows, which radios (currently off) must be turned on (and which channels must be assigned to them) and which radios (currently on) must be turned off. MP-CARA consists of two stages. In the first stage, channels are assigned to radios being turned on in order to maximize the network performance, while satisfying the node power constraints. In the second stage, traffic flows are routed such that the node power constraints are met and the total power consumption is minimized. We show that the latter problem can be formulated as the Fixed Charge Network Flow (FCNF) problem, which is NP-hard. The problem addressed in the second stage is thus solved by using a heuristic that finds a local optimum for the FCNF problem [2], adapting it to take the additional power constraints into account.

The remainder of this paper is organized as follows. An overview of the related work is given in Section II. In Section III we present a model for the node power consumption and formalize the minimum power channel assignment and routing problem. In Section IV we illustrate the operations of MP-CARA. The theoretical analysis developed to derive the maximum power consumption allowed to a node is presented in Section V. We present a performance evaluation of our heuristic in Section VI and conclude the paper in Section VII.

II. RELATED WORK

The minimization of the energy consumed in wireless networks has been the subject of a plethora of research work in the past years, mainly because devices in ad hoc and sensor networks typically operate on battery power, and hence minimizing the energy consumption is required to prolong the network lifetime. In this context, several techniques to minimize the energy consumption have been proposed in different areas: topology control [3], scheduling [4], routing [5]. Other work proposes a joint use of these techniques [6] or a cross-layer approach [7].

More recently, power saving techniques have been applied to wireless mesh networks. A few papers focus on solar powered mesh networks. In [8], the problem of dimensioning solar panels and batteries in order to avoid power outages is addressed, given historical solar insolation data and the expected traffic profile. Authors assume that an energy-aware routing protocol is used and propose a genetic algorithm to solve this provisioning problem. In [9], authors address the same provisioning problem, but they account for the efficiency loss due to sub-optimal positioning and orientation of the solar panels. A similar provisioning problem is addressed in [10], where the solution is evaluated in two distinct scenarios (mountainous regions like the Alps and sub-Saharan countries like Tanzania). In contrast to these previous works, we do not address a provisioning problem, but rather focus on optimizing channel assignment and routing.

The minimization of the energy consumption of a wireless mesh network is addressed in [11]. The authors assume a relationship between the transmission rate and the power consumption and simply minimize the sum of the power consumption over all the links. No attempt is made to switch off nodes and interference among links is not considered. The approach proposed in [12] aims at switching off as many nodes as possible while satisfying the throughput guarantees of the admitted flows. A routing metric that weighs battery-operated mesh clients and mesh routers in a different manner is proposed in [13], where the aim is to prolong the lifetime of a wireless mesh network deployed for emergency response in disaster-hit areas. All the approaches described so far consider single-radio WMNs and hence do not deal with the additional complexity introduced by the channel assignment problem.

A few papers consider multi-radio wireless mesh networks and take energy considerations into account. A distributed scheme to save power in multi-radio WMNs is presented in [14]. Each node attempts to select the lowest transmission rate that satisfies its throughput needs in order to save power. However, authors do not deal with the routing problem and no attempt is made to switch off nodes or radios. Another distributed scheme is presented in [15], which employs an identical channel assignment, i.e., all the nodes are assigned the same set of channels. Normally, only one radio is active, while the others are only activated when the IEEE 802.11e video queue occupancy is above a certain threshold. However, the routing problem is not addressed and it is known that the identical channel assignment is not the best choice. In [16], authors formulate two linear programming models to find the link scheduling, channel assignment and routing that maximize the network throughput with the minimum consumption of energy. However, authors assume a time-slot based channel access and require that links operate on different channels in distinct time slots. In [17], the same authors investigate the features of the solutions obtained while varying the number of available channels and radios.

The closest work to this paper is our previous approach proposed in [18]. Such work lacks a model for the node power consumption and hence the minimization of the energy consumption of a WMN is pursued by simply trying to turn off as many radios as possible. The model presented here is substantially more complete, as it accounts for the power consumed by a radio in the different states, and therefore leads to better performing configurations. Also, [18] focuses on WMNs where mesh nodes have a fixed power connection and therefore does not handle constraints on the power consumption.

III. PROBLEM FORMULATION

Network topology and collision domains: We model the WMN as a directed graph $G_I = (V, E_I)$, where $V$ is a set of nodes each representing a mesh router. Given two nodes $u, v \in V$, a directed edge $u \rightarrow v$ belongs to $E_I$ iff $v$ is able...
to successfully receive frames sent by $u$. Each mesh router $u$ is equipped with a set $\mathcal{R}(u)$ of radio interfaces and there are $|\mathcal{C}|$ orthogonal channels. Given that a radio may serve multiple links and the ability of commodity hardware to set the transmission rate on a per-packet basis, a rate is assumed to be assigned to each link rather than to the radio. The transmission rate used on link $u \to v$ denotes its capacity $c(u \to v)$. $G_I$ is referred to as the potential communication graph since an edge $u \to v \in E_I$ indicates that $u$ can transmit to $v$ provided that they are assigned a common channel. Finally, $v$ is a potential neighbor of $u$ ($v \in \mathcal{N}(u)$) if $\{u \to v, v \to u\} \subseteq E_I$.

A channel assignment $\mathcal{A}$ assigns a channel $\mathcal{C}(r) \in \mathcal{C}$ to each radio $r$ of every node $u \in V$. We denote by $\mathcal{A}(u)$ the set of channels assigned to radios of $u$. With this assignment, $\mathcal{A}$ induces a new graph model $G = (V,E)$ where two nodes $u$ and $v$ are connected if $u \to v \in E_I$ and they share at least one common channel. In case $u$ and $v$ share multiple channels, the set $E$ may include as many links between the two nodes as the number of common channels. To indicate that a link has been assigned channel $c$, we use the notation $u \xrightarrow{c} v$. Given a radio $r \in \mathcal{R}(u)$, we also denote by $\mathcal{L}'(r) = \{v \xrightarrow{c} u \in E\}$ and $\mathcal{L}^c(r) = \{u \xrightarrow{c} v \in E\}$ the set of incoming and outgoing links of radio $r$, respectively, and by $\mathcal{L}(r) = \mathcal{L}'(r) \cup \mathcal{L}^c(r)$. A link $x \xrightarrow{c} y \in E$ interferes with $u \xrightarrow{c} v \in E$ if a transmission on $x \xrightarrow{c} y$ prevents a simultaneous transmission on $u \xrightarrow{c} v$. We define the set of all the links that interfere with $u \xrightarrow{c} v$ as its collision domain and denote by $D(u \xrightarrow{c} v)$. In other words, none of the links in $D(u \xrightarrow{c} v)$ can be active at the same time as $u \xrightarrow{c} v$. Finally, we define the total utilization of the collision domain of link $e$ as $U_{\text{tot}}(e) = \sum_{e_0 \in D(e)} f(e_0)$, where $f(e_0)$ denotes the amount of flow routed on link $e_0$. The total utilization of a collision domain is an estimate of the fraction of time during which the wireless channel is busy due to transmissions on the links of the collision domain.

**Power consumption and energy harvesting model:** The average power consumed by a node $u$ in the sleep state is denoted by $P_S(u)$, while in the active state it is given by a baseline power consumption $P_B(u)$ plus the average power consumed by each radio interface. The latter is given by the sum of the power consumed in each of the possible radio states (transmitting, receiving, idle and sleep mode), each weighted by the probability of being (or, equivalently, the fraction of time spent) in that state, if the radio is active, and by the power consumption in the sleep state, otherwise [19]. Thus, the power consumed by an active node $u$ can be expressed as:

$$P(u) = P_B(u) + \left|\{r \in \mathcal{R}(u) \mid \mathcal{C}(r) = \emptyset\}\right| \cdot P_{\text{sleep}} + \sum_{r \in \mathcal{R}(u) \mid \mathcal{C}(r) \neq \emptyset} \left[\tau_{TX}(r)\rho_{TX} + \tau_{RX}(r)\rho_{RX} + \tau_{idle}(r)\rho_{idle}\right] P_{\text{sleep}}$$

(1)

where $\mathcal{C}(r) = \emptyset$ denotes that radio $r$ is in sleep mode, $\rho_{TX}$, $\rho_{RX}$, $\rho_{idle}$ and $\rho_{sleep}$ represent the power consumption of a radio in the transmitting, receiving, idle and sleep mode, respectively, and $\tau_{TX}(r)$, $\tau_{RX}(r)$ and $\tau_{idle}(r)$ are the fraction of time spent by radio $r$ in the transmitting, receiving and idle mode, respectively. We remark that the radio power consumption values ($\rho_{TX}$, $\rho_{RX}$, $\rho_{idle}$ and $\rho_{sleep}$) may vary from radio to radio, depending on the radio characteristics and on the transmission rate and power used. It is only for the sake of simplicity that such a dependence is not made explicit in the notations used. Finally, the fraction of time spent by a radio in the transmitting (receiving) mode can be estimated as the sum of the flow to capacity ratios over all the links that leave (enter) that radio:

$$\tau_{TX}(r) = \sum_{l \in \mathcal{L}(r)} \frac{f(l)}{c(l)}$$

$$\tau_{RX}(r) = \sum_{l \in \mathcal{L}(r)} \frac{f(l)}{c(l)}$$

$$\tau_{idle}(r) = 1 - \tau_{TX}(r) - \tau_{RX}(r)$$

(2)

Each node is powered by an energy harvester. As in [20], the harvested power in every time interval is modelled as a sequence of i.i.d. random variables. Each node $u$ is also equipped with a battery which stores, up to its capacity $B(u)$, the energy supplied by the energy harvester exceeding the amount required by the node to operate.

**Minimum power channel assignment and routing problem:** We are given a set $D$ of traffic demands, each represented by a pair of nodes $(a,b)$. The amount of flow associated with a demand $(a,b)$ is denoted by $F^{ab}$ and the total traffic load is denoted by $F = \sum_{(a,b) \in D} F^{ab}$. The amount of flow of a given demand $(a,b)$ routed on link $e$ is indicated by $f^{ab}(e)$, while the total amount of flow on link $e$ is denoted by $f(e)$. We are also given a set $\{\Pi(u)\}_{u \in V}$ specifying the constraints on the power consumption of the nodes.

The goal is to route the given set of traffic demands and assign channels to radios in such a way to deliver as much flow as possible to the destination, while minimizing the total power consumption of the whole network and satisfying the power constraint on every node. In [21], it has been shown that a necessary condition for every link $e$ of a collision domain to achieve a transmitted data rate $f(e)$ is that the total utilization of the collision domain is below a certain threshold $\lambda_0$, which depends on the overhead of the channel access function. Thus, to allow for the whole total traffic load to be delivered to the destination, the total utilization of every collision domain in the network needs to be below the threshold $\lambda_0$. In [21], it has been indeed verified through extensive simulations that: (i) as long as the maximum total utilization among all the collision domains is less than $\lambda_0$, there is a high probability ($\sim85\%$) that a high percentage ($>95\%$) of the traffic load is delivered to the destination; and (ii) if the maximum total utilization exceeds that threshold, the percentage of the traffic load that is delivered to the destination is a decreasing function of the maximum total utilization.

Such results suggest that, to maximize the portion of the total traffic load that is delivered (i.e., the throughput), we should strive to keep the maximum total utilization among all the collision domains below a given threshold $\lambda_0$ or, if this is not feasible, we should minimize the maximum total utilization. Based on these results, we formulate the following:

**Minimum Power Channel Assignment and Routing Problem:** Given the potential communication graph $G_I(V,E_I)$ of a WMN, a set of traffic demands $\{F^{ab}\}_{(a,b) \in D}$ and a set of node power constraints $\{\Pi(u)\}_{u \in V}$, determine how to assign channels to radios and route the traffic demands in such a way that $\sum_{u \in V} P(u)$ is minimized, subject to:
1) \( \max_{e \in E} U_{\text{tot}}(e) \) is the set of traffic demands \( \mathbb{G} \) in the following problem:

\[
\max_{e \in E} \sum_{e \in D(e)} c(e) \leq \max(U_{\text{tot}}^0, \lambda_0)
\]

2) \( \mathcal{P}(u) \subseteq \Pi(u) \quad \forall u \in V \)

where 2) imposes that the average power consumed by a node \( u \) does not exceed a certain threshold \( \Pi(u) \). Given the battery capacity and the probability distribution of the harvested power, such a threshold can be chosen to ensure that the outage probability is below a given value, as shown through the theoretical analysis presented in Section V. \( U_{\text{tot}}^0 \) is the minimum achievable value for the maximum total utilization, i.e., the objective value of the optimal solution to the following problem:

**Channel assignment and routing problem:** Given the potential communication graph \( G_I(V, E_I) \) of a WMN, a set of traffic demands \( \{P_{ab}\}_{(a,b) \in D} \) and a set of node power constraints \( \{\Pi(u)\}_{u \in V} \), determine how to assign channels to radios and route the traffic demands to minimize

\[
\max_{e \in E} \sum_{e \in D(e)} c(e) \quad \text{subject to} \quad \mathcal{P}(u) \subseteq \Pi(u) \quad \forall u \in V.
\]

The minimum power channel assignment and routing problem is NP-complete, since it has been shown in [1] that determining whether a channel assignment exists such that the maximum total utilization among all the collision domains is less than a given threshold is NP-complete. Hence, in the next section we propose a heuristic to solve this problem.

**IV. A MINIMUM POWER CHANNEL ASSIGNMENT AND ROUTING ALGORITHM**

**A. Assumptions**

MP-CARA, the heuristic we propose for the minimum power channel assignment and routing problem defined above, is a centralized algorithm and requires full knowledge of the current network status. In this subsection, we discuss ways for the central management station in charge of running the algorithm to collect the required information.

**Network topology.** To disseminate the information on the network topology (link set, transmission rates used on all the links), simple messages flooded throughout the network (like the TC messages provided by Optimized Link State Routing (OLSR)) can be employed. Alternatively, if the planar coordinates of the mesh nodes are known, a wireless link between two nodes can be assumed to exist if there is a transmission rate \( R \) such that the Signal-to-Interference and Noise Ratio (SINR) at the receiver (in the absence of transmissions on other links) exceeds the minimum SINR \( \gamma_R \) required to correctly decode a signal modulated at that rate. Thus, a link \( u \rightarrow v \in E_I \) if there exists a rate \( R \) such that

\[
G_{uv} P(u \rightarrow v) \geq \gamma_R, \quad P(u \rightarrow v) = \text{the power emitted by } u \text{ to transmit to } v, G_{uv} \text{ is the gain of the radio channel between } u \text{ and } v, \text{ and } n \text{ is a constant representing the thermal noise. The capacity of the link } c(u \rightarrow v) \text{ is assumed to be the highest rate } R \text{ for which such inequality holds. In this case, the transmission power and the antenna gain, if not fixed and known, can be disseminated through proper messages.}
\]

**Power consumption profile.** The power consumption values for each node \( \{P_S(u) \} \) and for each node \( \{P_{TX}, P_{RX}, \rho_{idle}, \rho_{sleep}\} \) can be derived from the products datasheets or measured experimentally. These values are fixed and hence do not need to be updated.

**Power constraints.** The set of constraints on the node power consumption are derived through the analysis presented in Section V, which requires the knowledge of the battery capacity, the maximum acceptable outage probability and the probability distribution of the harvested power. If a model of the latter is not available, it can be empirically determined from the statistics collected by the energy harvester.

**Interfering links.** The set of interfering links can be estimated through measurement techniques such as that proposed in [2]. Alternatively, the two-hop interference model [23], according to which two-hop neighboring links are not allowed to transmit at the same time, can be employed to estimate the set of interfering links. If the planar coordinates of the nodes are known, instead, a link \( x \rightarrow y \) can be assumed to interfere with \( u \rightarrow v \) if a transmission on \( x \rightarrow y \) makes the SINR at \( v \) too low to correctly decode the signal from \( u \) (this requires that transmission powers and antenna gains, if not fixed and known, are disseminated through the network).

**Traffic demands.** The set of traffic demands can be measured through techniques such as that proposed in [24].

**Channel assignment.** The channel assignment is computed by the central management station by running the proposed algorithm. Channels can be communicated to the mesh nodes by means of a simple protocol (e.g., the one employed in [25]).

**B. Design guidelines**

Maximizing the network performance while minimizing the total power consumption requires to adapt the network configuration to the offered traffic load. Indeed, more devices might be turned off if the traffic load decreases, while there might be the need to turn on some devices when the traffic load increases. Hence, MP-CARA has been designed to be run periodically (or every time the traffic load changes significantly). One of the design criteria followed in the definition of MP-CARA is to avoid changing channels on the radios that remain on after executing the algorithm. The reason is that, as shown in [25] through extensive experiments, switching channels on a number of radios causes a non-negligible interruption (in the order of 10 seconds) in the connectivity among neighbors and a more prolonged period during which routing loops and route instability occur. Thus, to avoid disrupting the network operation, MP-CARA takes the current channel assignment into account and abstinents from switching channels on radios.

MP-CARA aims to minimize the power consumption of the whole network, while ensuring that the constraint on the power consumption is met for every node and the maximum total utilization over all the collision domains is either below the given threshold \( \lambda_0 \) or is the minimum achievable one. MP-CARA consists of two stages, which are described next by means of a pseudo-code (Fig. 1).

**C. First stage**

In the first stage, traffic demands are routed and radios are turned on and assigned a channel in order to lower the maximum total utilization below the threshold \( \lambda_0 \), while ensuring that the power constraints are met for all nodes.
The rationale is that turning a radio on allows to establish new links, which can carry some flow away from collision domains with a high total utilization (thus helping to decrease the maximum total utilization) or from nodes with stringent power constraints (thus helping to meet the power constraints on such nodes). Note that no radio is turned off in this stage.

The first stage begins by putting into sleep mode all the nodes whose power constraints are so stringent to prevent them from being active (lines 1–3). We use the variable \( \max U^\text{cur}_t \) to keep the value of the maximum total utilization for the current solution. Initially, as no solution is available, such variable is set to infinity. In the attempt to find an initial solution, a linear program, denoted as MinMaxUTot-LP and described next, is solved (line 5). MINMAXUTOT-LP searches for a solution that minimizes the maximum total utilization among those that meet the power constraints for all the nodes, given the current channel assignment and the current set of nodes/radios turned on. If the model is feasible, the returned solution is stored as the current solution found by MP-CARA and its maximum total utilization is assigned to \( \max U^\text{cur}_t \). Otherwise (line 7), we solve MinMaxUTot-LP*, i.e., MinMaxUTot-LP without the power consumption constraints (constraints 2) and 3) in Fig. 2. Clearly, the returned solution cannot be used as the current solution found by MP-CARA, neither can the returned solution be used as the current solution found by MP-CARA, and its maximum total utilization be assigned to \( \max U^\text{cur}_t \) (which remains infinite). However, the returned solution provides a starting point to look for a feasible solution. Indeed, from that solution we can identify the nodes that do not meet the power constraints and take actions to reduce their traffic load.

If \( \max U^\text{cur}_t \) is above the threshold \( \lambda_0 \) (which includes the case when \( \max U^\text{cur}_t \) is still infinite, i.e., a solution that meets the power constraints for all the nodes has not been found), we turn some radios on in the attempt to find a solution with a better maximum total utilization or a solution that meets the power constraints for all the nodes. To this end, all the nodes having radios currently switch off are inserted into a priority queue \( Q \). The priority of a node depends on whether a solution that meets the power constraints for all the nodes has been found. If not, the priority of a node \( u \) is given by:

\[
\sum_{u \rightarrow v \in E} f(u \rightarrow v) \cdot \max \left( \frac{P(v)}{\Pi(v)}, 1 \right) \prod_{r \in \mathcal{R}(v)} \max(\tau_{TX}(r) + \tau_{RX}(r), 1)
\]

where the term in square brackets measures the extent to which the power constraint on a neighbor \( v \) is exceeded, multiplied by the sum of the fractions of time spent by each radio of \( v \) in transmitting and receiving nodes (if greater than 1).

Thus, higher priority is given to nodes that send large amount of traffic to nodes where the power constraints are farther from being met (according to the flow distribution returned by MinMaxUTot-LP*). The rationale is that, by turning on a radio of the extracted node, traffic can be diverted to the newly established links, thus alleviating the load on nodes where the power constraints are not met.

If a solution that meets the power constraints for all the nodes has been found, higher priority is given to nodes whose incident links contribute more (in terms of their flow to capacity ratio) to the total utilization of collision domains with \( U_{tot} > \lambda_0 \). Indeed, turning on a radio of a node leads to establishing new links which can carry some of the flow routed on preexistent links incident on that node (thus reducing the flow to capacity ratio of the latter). Since it is desirable to reduce the flow on those links that allow as many collision domains with \( U_{tot} > \lambda_0 \) as possible to benefit from such operation, each link \( l \) incident on a given node is weighed by its flow to capacity ratio times the number of links whose collision domain includes \( l \) and has a total utilization above \( \lambda_0 \). Following this, the priority of a node \( u \) is given by the sum of the weights of all its incident links:

\[
\sum_{e = x \rightarrow y \in E \mid x = u \vee y = u} \frac{f(e)}{c(e)} \cdot |\{e_0 \in E \mid e \in D(e_0) \wedge U_{tot}(e_0) > \lambda_0\}|
\]

Nodes are extracted one-by-one from \( Q \) in decreasing order

Fig. 1. MP-CARA pseudo-code

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of priority until $Q$ is empty or the current maximum total utilization is below $\lambda_0$. For each extracted node $u$, the channel to assign to the radio being turned on must be determined. For each channel $c$ that is not already assigned to any radio of $u$, we compute the set $S_u(c)$ of the new links that may be established. $S_u(c)$ consists of all the links $u \xrightarrow{v} v$ and $v \xrightarrow{u}$ such that $v$ is a potential neighbor of $u$ that either already has a radio on channel $c$ or has an available radio. For each candidate channel $c$, we solve a linear program (MAXFC($c$)):

$$\max \sum_{l=x \rightarrow y \in E} x_l - \sum_{l \in S_u(c)} x_l, \quad \text{s.t.} \sum_{u \rightarrow v \in E} x_l = \sum_{v \rightarrow u \in E} x_l,$$

where $x_l$ is the flow moved from an existing (to a new) link $l$. The rationale is as follows. In case a feasible solution has not been found yet, we want to maximize the decrease in the amount of time spent by the radios of the neighbors in transmitting (receiving) data over the links to (from) $u$, while minimizing the amount of time spent transmitting or receiving by the radio of the extracted node which is being turned on. Otherwise, we want to maximize the decrease in the total utilization of the collision domains including the existing links of $u$ while minimizing the impact on the total utilization of the new links. After evaluating all candidates, we select the channel $c^*$ with the highest optimal objective value. A radio on $u$ and on its neighbors (where needed) is turned on and assigned channel $c^*$ (line 13). All the links in $S_u(c^*)$, but those whose collision domain has a total utilization greater than $\max U^{\text{cur}}_{\text{tot}}$ even when they carry no flow, are then added to the network topology (line 14). MINMAXTOT-LP is solved to check whether the addition of the new links enables to find a better solution than the current one (if any). If a feasible solution has already been found, MINMAXTOT-LP is guaranteed to be feasible and to return a solution (which is stored as the new current solution) with a maximum total utilization not greater than $\max U^{\text{cur}}_{\text{tot}}$. If MINMAXTOT-LP is still infeasible after the addition of the new links, MINMAXTOT-LP$^*$ is solved in order to obtain a new configuration that can hopefully lead to a feasible solution (line 17). Before extracting a new node, the queue $Q$ is updated as follows: (i) if node $u$ has other radios that can be turned on, it is inserted into the queue $Q$ again, and (ii) all the neighbors of $u$ that no longer have available radios are removed from $Q$. The first stage ends when the maximum total utilization is below the threshold or there are no more radios to turn on.

**MINMAXTOT-LP:** Given the network topology, the channel assignment, the set of traffic demands and the node power constraints, we formulate a linear program (MINMAXTOT-LP, fig. 2) to route the traffic demands in such a way to minimize the maximum total utilization among all the collision domains while satisfying the power constraints. We introduce variable $M$, which is constrained to be greater than the total utilization of every collision domain (constraint 1). The goal of minimizing the maximum total utilization is thus achieved by minimizing $M$. Constraint 2 imposes that the fraction of time spent by each radio in the transmitting or receiving modes must not exceed 1. Constraint 3 represents the power constraints for all the active nodes. Constraint 4 represents the usual flow conservation constraint, while constraint 5 prevents the flow of a traffic demand from entering the source node or leaving the destination node. The returned solution, if any (constraints 2 and 3 might make the problem infeasible), meets the power constraints and has the minimum value for the maximum total utilization achievable with the given channel assignment.

**D. Second stage**

The second stage addresses the problem of routing the traffic demands and turning radios and nodes off in order to minimize the power consumption of the whole network, while ensuring not to increase the maximum total utilization beyond the maximum between the threshold $\lambda_0$ and the maximum total utilization achieved after the first stage. We first show...
that this problem is analogous to the well-known Fixed Charge Network Flow (FCNF) problem, and hence NP-hard [26], and then present a heuristic to find an approximate solution.

The FCNF problem is a minimum cost capacitated multi-commodity flow problem where the cost of an edge $e_j$ carrying $f_j$ units of flow is 0 if $f_j = 0$ or $c_i f_j + s_i$ otherwise ($c_i, s_i \geq 0$). Thus, the cost function has a discontinuity at the origin. The problem addressed in the second stage can be reformulated as the FCNF problem by means of the following graph transformation (Figs. 3a and 3b). Let $G = (V, E)$ be the directed graph representing the WMN as returned by the first stage and $G_T = (V_T, A_T)$ be the transformed graph. $V_T$ includes, for each node $u \in V$ turned on, a vertex $u$, a vertex $u^*$, and a vertex $r_{u,v}$. For each radio turned on ($c$ is the channel assigned to the radio), $A_T$ consists of a set $A_N$ of arcs of type node, a set $A_R$ of arcs of type radio and a set $A_L$ of arcs of type link. Two arcs of type node are added between every pair of vertices $u$ and $u^*$, while two arcs of type radio are added between every pair of vertices $u^*$ and $r_{u,v}$. The capacity of all these arcs is $F$. An arc of type link from $r_{u,v}$ to $r_{v,u}$ exists iff $u \rightarrow v \in E$ and it has the same capacity as the corresponding link. For ease of notation, we denote by $\bar{a}$ the inverse arc of an arc $a \in A_N \cup A_R$. We define the cost function $\gamma_a$ of each arc $a$ as follows:

$$
\gamma_a(z) = (\rho_{TX} + \rho_{RX} - 2\rho_{idle}) \cdot \frac{z}{c(a)}, \quad \forall a \in A_L
$$
$$
\gamma_a(x) + \gamma_a(y) = \gamma_{a,a}(x + y), \quad \forall a \in A_N \cup A_R
$$
$$
\gamma_{a,a}(z) = \begin{cases} 
\rho_{on,a} & \text{if } z \neq 0 \\
\rho_{off,a} & \text{if } z = 0 
\end{cases}
$$

where, if $a = u \rightarrow u^* \in A_N$ or $a = u^* \rightarrow u \in A_N$:

$$
\rho_{on,a} = P_B(u) + \rho_{sleep}, \quad \|\{r \in R(u) | C(r) = \emptyset\}\|
$$
$$
\rho_{off,a} = P_S(u) - \rho_{sleep}, \quad \|\{r \in R(u) | C(r) \neq \emptyset\}\|
$$

and, if $a \in A_R$:

$$
\rho_{on,a} = \rho_{idle}, \quad \rho_{off,a} = \rho_{sleep}
$$

Given that the average power consumed by an active radio $r$ can be expressed as:

$$
\tau_{TX}(r) \rho_{TX} + \tau_{RX}(r) \rho_{RX} + (1 - \tau_{TX}(r) - \tau_{RX}(r)) \rho_{idle} = \tau_{TX}(r) \cdot (\rho_{TX} - \rho_{idle}) + \tau_{RX}(r) \cdot (\rho_{RX} - \rho_{idle}) + \rho_{idle}
$$

it turns out that

$$
\Gamma(u) \overset{\text{def}}{=} \gamma_{u \rightarrow u^*}(f(u \rightarrow u^*)) + \gamma_{u^* \rightarrow u}(f(u^* \rightarrow u)) + \sum_{c \in A(u)} \gamma_{u^* \rightarrow r_{v,c}}(f(u^* \rightarrow r_{v,c})) + \gamma_{r_{v,c} \rightarrow u^*}(f(r_{v,c} \rightarrow u^*)) + \sum_{a \in A_L} (\rho_{TX} - \rho_{idle}) \cdot \frac{f(a)}{c(a)} + \sum_{a \in A_L} (\rho_{RX} - \rho_{idle}) \cdot \frac{f(a)}{c(a)}
$$

(5)

equals the power consumption $P(u)$ of a node $u$, and

$$
\sum_{a \in A_T} \gamma_a(f(a))\text{ equals the power consumption of the whole network. Hence, the problem of minimizing the total power consumption given the current channel assignment can be formulated as the MINPOWER problem of Fig. 4. Constraint 1 ensures that the maximum total utilization does not exceed the maximum between the maximum total utilization of the current solution and the threshold $\lambda_0$. Constraint 2 imposes that the fraction of time spent by each radio in the transmitting or receiving modes must not exceed 1. Constraint 3 represents the power constraints for all the active nodes. Constraints 4 to 8 represent the flow conservation constraints for each demand. In particular, constraints 4 and 5 impose that the sum of the flow on all the arcs of type link entering (leaving) a node $r_{u,v}$ equals the flow on the arc $r_{u,v} \rightarrow r_{v,u}$ (or $r_{v,u} \rightarrow r_{u,v}$). Constraints 6 and 7 impose that the sum of the flow on all the arcs of type radio entering (leaving) a node $u$ equals the flow on the arc $u^* \rightarrow u$ (or $u \rightarrow u^*$). Constraint 8 imposes the flow conservation.

Given the definition of the cost functions $\gamma_a$ (Fig. 5a) shows that the cost functions of arcs of type node or radio are discontinuous at the origin, it turns out that the MINPOWER problem is analogous to the FCNF problem, with additional constraints on the maximum total utilization and on the node power consumption, and hence NP-hard. Therefore, to find a solution to the MINPOWER problem, we employ an adapted version (to take the additional constraints into account) of one of the best heuristics known for the FCNF problem [2]. The main idea behind such heuristic is to approximate the
The algorithm stops when the same vector \( \vec{y} \) is returned at the end of the first stage and hence \( \Phi^r(\vec{u}) = \Pi(\vec{u}) \) since such solution is feasible. We note that, if we chose to set \( y_{a,\bar{a}}^{(0)} = 0 \) for some \((a,\bar{a})\) and \( f(a) + f(\bar{a}) > \varepsilon_{a,\bar{a}} \), then \( \phi^{a,\bar{a}}(f(a) + f(\bar{a})) = \phi_{0,a}^{a,\bar{a}}(f(a) + f(\bar{a})) > \rho_{a,\bar{a}}^{on} \) and hence \( \Phi^r(\vec{u}) \) could exceed the power consumed by \( u \), for some node \( u \), thus making it not guaranteed that \( \Phi^r(\vec{u}) \leq \Pi(\vec{u}) \) and that the solution returned at the end of the first stage is feasible for \( \text{MINPOWER}_\varepsilon(\vec{y}) \).

Now we show that, if the \((m-1)\)-th instance of \( \text{MINPOWER}_\varepsilon(\vec{y}) \) is feasible, then also the \( m \)-th instance is feasible. Indeed, we show that a solution \( \{f(a)\}_{a \in A} \) to the \((m-1)\)-th instance is also feasible for the \( m \)-th instance.

We recall that a solution to the \((m-1)\)-th instance determines the values of \( \vec{y}^{(m)} \) which are used as input for the next instance. Given that two instances of \( \text{MINPOWER}_\varepsilon(\vec{y}) \) differ in some \( y_{a,\bar{a}} \) values and hence in the expression of some cost functions \( \phi^{a,\bar{a}}(z) \), we only need to check that a solution to the \((m-1)\)-th instance satisfies the power constraints of the \( m \)-th instance. For each pair \((a,\bar{a}) \in A_N \cup A_R \) such that \( y_{a,\bar{a}}^{(m)} \neq y_{a,\bar{a}}^{(m-1)} \), two cases can occur:

- \( y_{a,\bar{a}}^{(m)} = 1 \) and \( f(a) + f(\bar{a}) < \varepsilon_{a,\bar{a}} \Rightarrow y_{a,\bar{a}}^{(m)} = 0 \): 
  \[ \phi_{0,a}^{a,\bar{a}}(f(a) + f(\bar{a})) < \phi_{1,a}^{a,\bar{a}}(f(a) + f(\bar{a})) \]
- \( y_{a,\bar{a}}^{(m)} = 0 \) and \( f(a) + f(\bar{a}) \geq \varepsilon_{a,\bar{a}} \Rightarrow y_{a,\bar{a}}^{(m)} = 1 \):
  \[ \phi_{1,a}^{a,\bar{a}}(f(a) + f(\bar{a})) < \phi_{0,a}^{a,\bar{a}}(f(a) + f(\bar{a})) \]

Thus, \( \phi^{a,\bar{a}}(f(a) + f(\bar{a}))^{(m)} \leq \phi^{a,\bar{a}}(f(a) + f(\bar{a}))^{(m-1)} \) \( \forall (a,\bar{a}) \in A_N \cup A_R \) in both cases and hence \( \Phi^r(\vec{u})^{(m)} \leq \Phi^r(\vec{u})^{(m-1)} \leq \Pi(\vec{u}) \). Thus, the \( m \)-th instance of \( \text{MINPOWER}_\varepsilon(\vec{y}) \) is feasible.
\(\varepsilon(\bar{y})\) is feasible as well. By induction, it follows that all the instances of \text{MINPOWER-}\varepsilon(\bar{y})\) are feasible.

When the algorithm ends, the solution to the last solved instance of \text{MINPOWER-}\varepsilon(\bar{y})\) is such that \(f(a) + f(\bar{a}) \in \{0\} \cup [\varepsilon_a, \bar{a}, +\infty)\) for every pair of arcs \((a, \bar{a})\) of type node or radio. Given that \(\phi^{a_{\bar{a}}}(z) = \gamma_{a, \bar{a}}(z) \quad \forall z \in \{0\} \cup [\varepsilon_a, \bar{a}, +\infty)\), it turns out that \(\Phi^u(\bar{u}) = \Gamma u \quad \forall u \in V\) and hence the solution to the last solved instance of \text{MINPOWER-}\varepsilon(\bar{y})\) is also a solution to \text{MINPOWER}.

The above theorem allows to conclude that, if the first stage returns a feasible solution, the second stage also returns a feasible solution. Such a solution is obtained from the last solved instance of \text{MINPOWER-}\varepsilon(\bar{y})\) by turning off a node \(u\) if \(f(u \rightarrow u^*) + f(u^* \rightarrow u) = 0\) and the radio on node \(u\) that was set to channel \(c\) at the end of the first stage if \(f(u^* \rightarrow r^{u, c}) + f(r^{u, c} \rightarrow u^*) = 0\) (lines 36–37 in Fig. 1).

V. A THEORETICAL ANALYSIS OF THE OUTAGE PROBABILITY

MP-CARA requires as input the set of constraints on the node power consumption. As shown in this section, such constraints can be computed in such a way to ensure that the outage probability is below a certain value. Indeed, we next present a theoretical analysis that relates the battery capacity \(B\), the probability distribution of the harvested power and the constraint \(\Pi\) on the power consumption of a node to the outage probability of that node. We denote by \(\mathbf{H}\) the random variable representing the harvested power in a time interval of duration \(\Delta\). As in, e.g., [20], we consider a discrete battery with \(N + 1\) energy levels \(\{0, \varepsilon, 2\varepsilon, \ldots, N\varepsilon\}\), where \(\varepsilon\) is the minimum energy unit and \(B = N\varepsilon\) is the capacity of the battery. The energy level of the battery at the end of a time interval depends on the energy level at the beginning of the time interval and the difference between the harvested power and the consumed power. MP-CARA ensures that the latter is upper bounded by \(\Pi\). Thus, in order to conduct a worst case analysis, we assume that the power consumed in every time interval equals \(\Pi\). Given the independence of the random variables representing the harvested power in distinct time intervals, the energy levels can be considered as the states of a time-homogeneous Markov chain with transition matrix:

\[
P = \begin{pmatrix}
0 & \sum_{i=0}^{\infty} p_i & p_1 & p_2 & \cdots & p_{N-1} & \sum_{i=0}^{\infty} p_i & 1 \\
\sum_{i=0}^{\infty} p_i & p_0 & p_1 & p_2 & \cdots & p_{N-2} & \sum_{i=0}^{\infty} p_i & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & \sum_{i=0}^{\infty} p_i & p_{N-1} & p_{N-2} & \cdots & p_{-1} & \sum_{i=0}^{\infty} p_i & 0 \\
\end{pmatrix}
\]

where \(p_i = Pr(\mathbf{H} = i\varepsilon)\), \(i \in \mathbb{Z}\), is the probability that the difference between the harvested energy and the consumed energy in a time interval equals \(i\varepsilon\) energy units. Given that \(\mathbf{H}\) is a continuous random variable, we consider the following approximation (see Fig. 6):

\[
p_i = Pr\left(\left(1 - \frac{1}{2}\right)\varepsilon \leq \mathbf{H} - \Pi \leq \left(i + \frac{1}{2}\right)\varepsilon \right) = F_\mathbf{H}(\Pi + \left(i + \frac{1}{2}\right)\varepsilon) - F_\mathbf{H}(\Pi + \left(i - \frac{1}{2}\right)\varepsilon)
\]

where \(F_\mathbf{H}(\cdot)\) is the cumulative distribution function of \(\mathbf{H}\).

In order to simplify calculations, we choose the interval duration \(\Delta\) such that the energy consumed in a time interval is at most one energy unit, so that the energy level cannot decrease of more than one unit in a time interval (i.e., \(p_i = 0\) for \(i < -1\)). Using the approximation of (7), this condition leads to the following setting for \(\Delta\):

\[
\Pi\Delta = \frac{3}{2}\varepsilon = \frac{3}{2}\frac{B}{N} \Rightarrow \Delta = \frac{3}{2}\frac{B}{N\Pi}
\]

With the above setting of \(\Delta\), the transition matrix of the Markov chain becomes:

\[
P = \begin{pmatrix}
p_{-1} + p_0 & p_1 & p_2 & \cdots & p_{N-1} & \sum_{i=0}^{\infty} p_i & 1 \\
p_{-1} & p_0 & p_1 & p_2 & \cdots & p_{N-2} & \sum_{i=0}^{\infty} p_i & 0 \\
0 & p_{-1} & p_0 & p_1 & \cdots & p_{N-3} & \sum_{i=0}^{\infty} p_i & 0 \\
0 & 0 & p_{-1} & p_0 & \cdots & p_{N-4} & \sum_{i=0}^{\infty} p_i & 0 \\
0 & 0 & 0 & 0 & \cdots & p_{-1} & \sum_{i=0}^{\infty} p_i & 0 \\
\end{pmatrix}
\]

This Markov chain is irreducible, since (if \(p_{-1} > 0\)) any state can be reached from any other state with non-zero probability. Also, state 0 is aperiodic because of the self-transition having non null probability \(p_{-1} + p_0 > 0\). This is sufficient to prove that the Markov chain is ergodic and hence admits a unique steady-state probability distribution, which the chain converges to regardless of the initial state. Such limiting distribution \(\pi = (\pi_0, \pi_1, \ldots, \pi_N)\) can be found by solving the system of equations \(\pi = \pi P\) with the normalization constraint \(\sum_{i=0}^{N} \pi_i = 1\):

\[
\begin{pmatrix}
1-p_{-1} & -p_{-1} & 0 & \cdots & 0 & 0 & 0 \\
-p_{-1} & 1-p_0 & -p_{-1} & \cdots & 0 & 0 & 0 \\
-p_{-1} & -p_0 & 1-p_0 & \cdots & 0 & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
-p_{N-1} & -p_{N-2} & -p_{N-3} & \cdots & 1-p_0 & -p_{-1} & 0 \\
\end{pmatrix}
\begin{pmatrix}
\pi_0 \\
\pi_1 \\
\pi_2 \\
\cdots \\
\pi_{N-1} \\
\pi_N \\
\end{pmatrix}
= \begin{pmatrix}
0 \\
0 \\
0 \\
\cdots \\
0 \\
1 \\
\end{pmatrix}
\]

If we denote by \(A\) and \(b\), respectively, the matrix of coefficients and the vector of constant terms of the above system of equations, it turns out that \(\pi = A^{-1}b\). Thus, the outage probability \(\pi_0\) can be computed as:

\[
\pi_0 = (A^{-1})_{1,N+1} = \frac{C_{N+1,1}}{\det[A]} = \frac{(-1)^{N+2}(-1)^N(p_{-1})^N}{\det[A]} = \frac{(p_{-1})^N}{\det[A]}
\]
where $C_{i,j}$ is the $(i,j)$ cofactor. This result follows from observing that removing the first column and the last row from $A$ yields a triangular matrix whose determinant (i.e., the $(N+1,1)$ minor) is the product of the diagonal entries. We use the Doolittle method to find an LU decomposition of $A$, which allows us to easily compute the determinant of $A$:

$$A = \begin{pmatrix}
1 & 1 & \cdots & 1 \\
l_{1,1} & l_{2,1} & \cdots & l_{N,1} \\
l_{1,2} & l_{2,2} & \cdots & l_{N,2} \\
\cdots & \cdots & \cdots & \cdots \\
l_{1,N} & l_{2,N} & \cdots & l_{N,N}
\end{pmatrix} \begin{pmatrix}
u_0 & -p_{-1} & 0 & \cdots & 0 \\
u_1 & -p_{-1} & 0 & \cdots & 0 \\
u_2 & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
u_N & \cdots & \cdots & \cdots & \cdots
\end{pmatrix}$$

where:

$$u_i = \begin{cases}
1 - p_0 - p_{-1} & \text{if } i = 0, \\
1 - p_0 - \sum_{j=1}^{i-1} \frac{(p_0)^j}{\prod_{k=1}^{i-1} w_k} p_j & \text{if } 1 \leq i < N, \\
1 + \sum_{j=1}^{N} \frac{(p_0)^j}{\prod_{k=N-j}^{N} w_k} & \text{if } i = N.
\end{cases}$$

Hence:

$$\det[A] = \prod_{i=0}^{N} u_i = (p_0)^N + \sum_{i=0}^{N-1} (p_0)^{i-1} \prod_{j=0}^{i} u_j$$

as can be seen by multiplying $u_N$ by $\prod_{i=0}^{N-1} u_i$. Finally, let $x_i = \prod_{j=0}^{i} u_j$, it turns out that:

$$\pi_0 = \left(1 + \sum_{i=0}^{N-1} \frac{x_i}{(p_0)^{i+1}}\right)^{-1}$$

(9)

and $\{x_i\}_{i=0}^{N}$ is defined by the following recurrence relation:

$$x_i = \begin{cases}
1 & \text{if } i = -1, \\
1 - p_0 - p_{-1} & \text{if } i = 0, \\
(1 - p_0)x_{i-1} - \sum_{j=1}^{i-1} (p_0)^j p_j x_{i-j-1} & \text{if } i \geq 1.
\end{cases}$$

(10)

Equations (7), (8), (9) and (10) enable to compute (in $O(N^2)$ time) the outage probability given $B$, $\Pi$ and the probability distribution of $H$. By imposing the desired power outage probability, we can thus derive the power constraint $\Pi$ that yields this probability. It should be noted, however, that too stringent power constraints might make the minimum power channel assignment and routing problem infeasible. Hence, running MP-CARA is required to ensure that a feasible solution exists given the selected power constraints for all the nodes. If a feasible solution exists, it guarantees that the outage probability for each node does not exceed the value derived from the analysis presented in this section.

VI. PERFORMANCE EVALUATION

We conduct a number of simulation studies to compare MP-CARA to $E^2$CARA-TD [18] (an algorithm aiming to turn off as many radios and nodes as possible) and FCRA [1] (an algorithm aiming to minimize the maximum total utilization) in terms of maximum total utilization, total power consumption (according to the model of Section III), network throughput and energy consumption (measured from simulations). In a first series of simulations, we assume that all the nodes are supplied with enough power, in order to make a fair comparison with $E^2$CARA-TD and FCRA, which do not account for node power constraints. In the second series of simulations, we consider more stringent node power constraints to show that a power constraints-aware algorithm such as MP-CARA achieves much better results than power constraints-unaware algorithms coupled with a routing protocol that finds alternative paths whenever a node fails due to a power outage.

In both cases, the threshold $\lambda_0$ has been set to 0.50. We use WiMesh [27], a software tool we developed$^1$ to compute the network configuration returned by a channel assignment and routing algorithm and to automatically setup and run packet level simulations via the network simulator ns-3. In all experiments, we compute the network configuration (channel assignment and flow allocation) returned by each of the algorithms under test and evaluate the maximum total utilization and the node power consumption (eq. (1)). The throughput and the energy consumption achieved with each such network configuration are instead measured by running ns-3 simulations lasting 120 seconds. Two different routing protocols for the ns-3 simulations are considered in this paper: the default path selection protocol defined by the IEEE 802.11s amendment (HWMP – Hybrid Wireless Mesh Protocol) and the MPLS splitting forwarding paradigm presented in [28], according to which the incoming traffic (of a given demand) at an intermediate node is split among multiple next hops in proportion to predefined coefficients. Such coefficients are determined such that the traffic demands are routed according to the flow allocation returned by the channel assignment and routing algorithm. The energy model implemented by ns-3 provides that the power consumed by a radio is given by the product of the supply voltage (which is fixed to 3.0V) and the current intensity, which depends on the state in which the radio is. The current intensities values we use for the different states lead to the power consumption values reported in [29] and shown in Table I. The physical layer is IEEE 802.11a.

A. Comparison to other heuristics (loose constraints)

The aim of this set of simulations is to show that, in case all the mesh nodes are supplied with enough power, MP-CARA outperforms both an algorithm designed to turn off as many radios and nodes as possible ($E^2$CARA-TD) and an algorithm designed to maximize the network throughput (FCRA). We consider 128 distinct configurations determined by all the combinations of the following parameters:

- three connected topologies (50-, 75- and 100-nodes) where each node has two radios and a 75-nodes topology where nodes have either two or three radios
- the availability of 3 or 11 orthogonal channels
- two interference models (SINR-based and two-hop)
- two different sets of source-destination node pairs
- four different sets of traffic demands

For each configuration, the channel assignment computed by FCRA is passed as the initial configuration to the other two algorithms. Then, we consider three consecutive variations in the set of traffic demands. For each such variation, MP-CARA and $E^2$CARA-TD take as input the current channel

$^1$WiMesh is freely available at http://wpage.unina.it/stavallo/WiMesh/
as expected, the lower the traffic load, the higher the ratio of the throughput normalized to the sum of the traffic demands. As Section III, achieved by MP-CARA and E\textsuperscript{2}CARA-TD, in terms of maximum total utilization, while achieving substantial power savings (see below).

Maximum total utilization (Fig. 7a): As expected, the maximum total utilization decreases (increases) when the traffic load decreases (increases). MP-CARA achieves lower values than E\textsuperscript{2}CARA-TD and FCRA in all the scenarios. As for the median values, MP-CARA enables a maximum total utilization reduction from 4% to 8% with respect to E\textsuperscript{2}CARA-TD and from 12% to 19% with respect to FCRA. This shows that MP-CARA substantially outperforms E\textsuperscript{2}CARA-TD and FCRA in terms of maximum total utilization, while achieving substantial power savings (see below).

Number of nodes and radios turned off (Figs. 7c-7d): The figures show that MP-CARA turns off the highest number of nodes and radios both initially and after every variation in the traffic demands. MP-CARA is much more effective than E\textsuperscript{2}CARA-TD in turning off nodes (from 20% to 60% more nodes turned off in the four scenarios). FCRA, instead, independently of the traffic load, never turns off a node, and on average it turns off just a couple of radios.

Power consumption (Fig. 7b): The figure shows the power consumption, computed according to the model of Section III, achieved by MP-CARA and E\textsuperscript{2}CARA-TD normalized to that achieved by FCRA. Substantial power savings are enabled by MP-CARA (from 19% to 27% less power consumed on average) and by E\textsuperscript{2}CARA-TD (from 16% to 21% less power consumed on average) with respect to FCRA. Also, MP-CARA consumes less power than E\textsuperscript{2}CARA-TD in every considered scenario.

Throughput (Figs. 7e,7f): The figures show the achieved throughput normalized to the sum of the traffic demands. As expected, the lower the traffic load, the higher the ratio of the

![Fig. 7. Comparison among heuristics (loose power constraints)](image)

### TABLE I

VALUES FOR THE POWER MODEL (WATTS)

<table>
<thead>
<tr>
<th>$P_{id}$</th>
<th>$P_{TX}$</th>
<th>$P_{RX}$</th>
<th>$P_{idle}$</th>
<th>$P_{sleep}$</th>
<th>$P_{S}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.29</td>
<td>2.37</td>
<td>1.10</td>
<td>0.94</td>
<td>0.29</td>
<td>0.5</td>
</tr>
</tbody>
</table>

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traffic load that is delivered by the considered algorithms. We note that the performance of MP-CARA in terms of throughput is not affected by the high number of nodes and radios turned off. Indeed, the ratio of the traffic load that is delivered to the destinations by MP-CARA on average is higher than FCRA (6% to 16% for UDP, 5% to 10% for TCP) and $E^2$CARA-TD (10% to 16% for UDP, 6% to 10% for TCP).

**Energy consumption per Mbit (Figs. 7g,7h):** The figures show the energy consumed by each algorithm normalized to the amount of data (Mbits) it delivers to the destination, which can be seen as a measure of the energy efficiency of the algorithm. Delivering a megabit of data when using MP-CARA requires much less energy with respect to both FCRA (23% to 36% for UDP, 24% to 31% for TCP) and $E^2$CARA-TD (11% to 22% for UDP, 10% to 16% for TCP).

The results of this simulation study show that MP-CARA clearly outperforms the other algorithms, since, for different traffic load conditions, it is the most efficient algorithm (in terms of energy consumed per Mbit) while achieving the highest throughput. Also, the running times of MP-CARA are comparable to those of FCRA (from tens to few seconds on an Intel Core i7-3770 processor with 8GB RAM) and are one order of magnitude shorter than those of $E^2$CARA-TD.

### B. Comparison to other heuristics (stringent constraints)

We now consider the case in which not all the mesh nodes are supplied with enough power, in order to: (i) verify that MP-CARA enables to limit power outages; (ii) show that MP-CARA outperforms the other algorithms that do not consider the node power constraints, both when a reactive routing protocol (HWMP) is employed and when the MPLS splitting forwarding paradigm is employed.

We start from the same initial set of 128 configurations as in the previous simulation study and consider two consecutive traffic variations (as described in the previous subsection), each decreasing the traffic load of 40% on average. The mean value of the i.i.d. random variables representing the harvested power in every time interval is determined as follows. For one fifth of the mesh nodes, the mean harvested power varies uniformly between 40% and 60% of the maximum required power (given by eq. (1) assuming that all the radios are always in the transmitting mode), while for the remaining nodes the mean harvested power varies uniformly between 80% and 100% of the maximum required power. Two distinct sets of mean harvested power values are derived in such a way. For each set and each configuration, two distinct ns-3 simulations are conducted, differing in the probability distribution (uniform or exponential) of the random variables providing the amount of power harvested in every time interval. For each node, the constraint on the power consumption required by MP-CARA is set to the mean value of the random variable representing the power harvested by the associated energy harvester. Each node is equipped with a battery having a capacity of 10 Joules, which is fully charged initially and is periodically (once a second) recharged by the attached energy harvester. With such settings, the theoretical analysis of Section V yields a worst case outage probability of 0.05 in the case of uniform distribution and 0.16 in the case of exponential distribution. The results of this simulation study are summarized in Fig. 8.

**Power outages (Figs. 8a,8b):** The figures show the amount of time (seconds) spent altogether by the nodes in the sleep state due to power outages (the nodes turned off by the algorithms are not considered). It turns out that MP-CARA, in conjunction with both the MPLS splitting forwarding paradigm and HWMP, causes minimal power outages (within the bounds derived from the theoretical analysis of Section V). Instead, FCRA and $E^2$CARA-TD cause a certain percentage of the nodes (respectively, 20% and 10% on average) to be put in sleep mode for a period of time varying from a couple of seconds to 70 seconds (i.e., the 60% of the simulation time) due to depletion of energy.

**Throughput (Figs. 8c,8d):** The figures show the achieved throughput normalized to the sum of the traffic demands. As expected, the power outages experienced by nodes when the network is configured by FCRA or $E^2$CARA-TD negatively affect the performance in terms of throughput. Indeed, in both
traffic scenarios, the highest throughput is achieved by MP-CARA in conjunction with the MPLS splitting forwarding paradigm, which routes the demands according to the flow allocation returned by MP-CARA. With regards to the median values, MP-CARA with MPLS splitting allows a throughput increase of 80% to 140% for UDP (90% to 130% for TCP) with respect to the other algorithms with MPLS splitting, and a throughput increase of 110% to 220% for UDP (190% to 270% for TCP) with respect to the other algorithms with HWMP. We note that the MPLS splitting forwarding paradigm performs better than HWMP. This is because MPLS splitting employs multiple paths between a source-destination pair and routes the traffic demands according to the flow allocation returned by the algorithms (which minimizes the maximum total utilization given the selected channel assignment). On the contrary, HWMP is a single path routing protocol and disregards the flow allocation returned by the algorithms.

**Energy consumption per Mbit (Figs. 8e,8f):** The figures show the energy consumed by each algorithm normalized to the amount of data (Mbits) it delivers to the destination. It turns out that, in both traffic scenarios, MP-CARA in conjunction with the MPLS splitting forwarding paradigm is the most efficient combination. With regard to the median values, MP-CARA with MPLS splitting provides an energy consumed per Mbit improvement of 40% to 60% for UDP (40% to 65% for TCP) with respect to the other algorithms with MPLS splitting, and an improvement of 40% to 80% for UDP (55% to 80% for TCP) with respect to the other algorithms with HWMP.

**VII. CONCLUSIONS**

In this paper we addressed the minimum power channel assignment and routing problem. The goal is to minimize the power consumption of multi-radio wireless mesh networks where nodes have constraints on their power consumption, while guaranteeing the optimal network performance. Given the NP-hardness of this problem, we formulated a heuristic that routes the given traffic demands, turns off some radios and turns on some other radios (by assigning them a channel) in the attempt to find the best solution possible. Simulation studies showed that the proposed algorithm outperforms our previous proposal and a non-power-aware channel assignment algorithm. Furthermore, we presented a theoretical analysis to derive the outage probability of a node as a function of the maximum allowed power consumption, the battery capacity and the probability distribution of the harvested power.

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