Opportunistic Finite Horizon Multicasting of Erasure-coded Data

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Abstract—We propose an algorithm for opportunistic multicasting in wireless networks. Whereas prior multicast rate adaptation schemes primarily optimize long-term throughput, we investigate the finite horizon problem where a fixed number of packets has to be transmitted to a set of wireless receivers in the shortest amount of time – a common problem, e.g., for software updates or video multicast. In the finite horizon problem, the optimum rate critically depends on the recent reception history of the receivers and requires a fine balance between maximizing overall throughput and equalizing individual receiver throughput. We formulate a dynamic programming algorithm that optimally solves this problem. We then develop two low complexity heuristics that perform close to the optimal solution and are suitable for practical online scheduling. We further analyze the performance of our algorithms by means of simulation. They substantially outperform existing solutions based on throughput maximization or favoring the user with the worst channel, and we obtain a 30% performance improvement over the former and a 120% improvement over the latter in scenarios with Rayleigh fading. We further analyze the performance of the schemes under imperfect state information and observe an even higher improvement over the benchmark schemes.

Index Terms—Finite horizon multicast, wireless multicast, opportunistic multicast scheduling, completion time, imperfect feedback

1 INTRODUCTION

In recent years, multicasting data to mobile users (e.g., for the purpose of video streaming, video conferencing, IPTV, distribution of news and alerts, or application and operating system updates) has gained in importance. As an example, the most recent mobile network architecture, LTE, includes the evolved Multimedia Broadcast Multicast Service (eMBMS) specifically for the purpose of distributing data and mobile TV content in a cellular network. Since the amount of such traffic in cellular networks is increasing rapidly and wireless resources are scarce and costly, improving the efficiency of wireless multicast is of high practical relevance.

The most common method for wireless multicasting is broadcasting. The base station (BS) transmits at some fixed low rate or the rate supported by the worst receiver to ensure that all receivers are able to receive the multicast transmission. This exploits the wireless broadcast gain, allowing a single transmission to simultaneously serve all receivers. Opportunistic multicast scheduling (OMS) improves over plain broadcast by exploiting multiuser diversity [1]. Having the BS transmit at a rate higher than the broadcast rate to the subset of receivers that can receive at this rate may improve overall throughput and minimize broadcast delay [2].

The intuition is that in an environment with variable channels, receivers that are not served in one slot due to bad channel conditions will be served in later slots when their conditions improve, and thus over time all receivers will eventually receive all the data. Hence, there is a tradeoff between multicast gain and multiuser diversity gain. Specifically, whenever the BS transmits a packet, it is necessary to select a suitable transmit rate, i.e., modulation and coding scheme (MCS). The MCS determines the amount of data transmitted per time slot as well as the packet loss probability. More robust MCSs transport less data and are more likely to be decodable.

Selecting the transmission rate and thus the subset of receivers to multicast to is a complex problem that has been the focus of a range of OMS algorithms [3]. To simplify the scheduling problem and improve performance when multicasting data, such algorithms often use erasure codes that ensure that with high probability, each packet received by a receiver is useful (unless the receiver already decoded all data) [4], i.e., the identity of the received packets is unimportant. Fixed rate LDPC [5] or rateless LT or Raptor codes [6] are examples of such erasure codes that work well in practice.

Most of the existing OMS algorithms [1], [2], [4], [7]–[9] consider the infinite-horizon multicast problem, where the sender has an infinite number of packets to send. The goal of the optimization is thus to maximize the long-term throughput to all receivers. This setting is a good approximation for the case of multicasting very large files [10]. In the infinite-horizon problem, the average channel quality seen by an individual receiver is likely to be close to the actual average of the channel distribution (law of large numbers). Therefore, it is unlikely to
see large differences in the number of received packets among the receivers (over a sufficiently large amount of time) and the state of the system in terms of the number of received packets can be neglected.

In practice, however, multicasting data with a size on the order of only hundreds or several thousands of packets is much more common, particularly for mobile networks. For example, mobile applications, news, and operating system updates often have a size of hundreds of kilobytes to several megabytes. Also, when streaming video, it is common to apply erasure coding to blocks consisting of one or several groups of pictures (GOP) [11]. A GOP usually comprises a few hundred packets, depending on the video rate. Such block sizes are a suitable tradeoff between coding efficiency and playout delay (caused by the decoding delay).

In the common finite-horizon multicast problem considered in this paper, the main objective is to minimize the completion time, i.e., the time needed for all receivers to receive enough data to decode the current block. The relevant optimization criterion is thus the throughput achieved over the duration of a block, rather than the long-term average throughput. Depending on the scenario, algorithms that are optimal for the infinite-horizon problem may be far from optimal for the finite-horizon case, as shown in this paper.

The finite horizon multicast problem is inherently more complex than the infinite horizon counterpart. When multicasting an infinite amount of data among a homogeneous group of receivers (i.e., with the same average channel conditions), the optimum tradeoff between multiuser diversity and multicast gain only depends on the number of receivers and their current channel conditions. In expectation, differences in the amount of data received by the different receivers will even out over time and therefore do not have to be taken into account. In contrast, the optimum decision in the finite horizon case depends on the amount of data received thus far. More accurately, it depends on the amount of data each receiver still needs to obtain in order to decode the full block of data and thus complete the reception. Intuitively, in case a receiver lags behind in the reception when many other receivers are also still far from completing the block, the lagging receiver may catch up by itself and jointly maximizing throughput for all receivers may be the optimum decision. If, however, all other receivers are close to completion, optimizing the MCS only for the lagging receiver may be the optimum choice to minimize overall completion time, as all other receivers are likely to complete before the lagging receiver in any case. In summary, having to take into consideration the receiver state in the optimization makes it much more challenging to find optimal solutions for the finite-horizon multicasting problem.

The main contributions of our paper are as follows:

- We formalize the finite horizon OMS problem and propose a dynamic programming (Dyn-Prog) based solution that optimally adapts the MCS to minimize the completion time, the time at which all receivers have successfully received the required amount of data.
- The high complexity of Dyn-Prog renders this approach unsuitable for many practical scenarios. We thus propose a very simple state-based heuristic that selects the MCS that maximizes the instantaneous throughput of the receiver with the lowest number of packets received so far (called Max-Min).
- We further design a more complex adaptive algorithm that selects the MCS that results in the expected future system state that has the lowest expected completion time. We estimate completion time using a weighted Euclidean distance metric. The corresponding weighted completion time algorithm (Weighted-CT) measures the distance between the different possible future states and the final state where all receivers completed, with weights based on average throughput estimates of the receivers. To deal with imperfect state information, we further design an estimation-based version of the algorithm, Weighted-CTe. It estimates the distribution of the current channel state based on outdated past feedback information and predicts the evolution of receiver states.
- We compare the performance of our low-complexity heuristics to the optimal Dyn-Prog solution as well as to existing approaches that greedily maximize the throughput for all receivers and a broadcasting scheme that always transmits to all receivers. We analyze scenarios with homogeneous and heterogeneous receiver sets under a basic multi-state channel model as well as Rayleigh fading. Under Rayleigh fading and with up to 16 users, the Max-Min algorithm provides a performance gain of 95% over the broadcasting scheme and a gain of 15% over the throughput maximization scheme. At a slight increase in complexity, the Weighted-CT heuristic performs very close to the optimal Dyn-Prog strategy, with performance gains of 120% and 30% over the broadcast and throughput maximization schemes, respectively. For scenarios with more than 16 users the achieved gains are even higher. In scenarios with imperfect information, Weighted-CTe achieves gains of up to 130% and 60% over the prior schemes in homogeneous and heterogeneous scenarios, respectively. We further implement an energy model for wireless multicasting and analyze the energy efficiency of the different algorithms.

Our paper is organized as follows. Section 2 reviews prior work. In Section 3 we discuss our system model, including the channel model and the MCS dependent packet loss model. In Section 4, we provide the dynamic programming-based optimal scheduling algorithm, together with a basic example to provide some intuition into how the algorithm trades off receiver throughput depending on the system state. To address the problem of state space explosion and high complexity of the dynamic programming solution, we propose two low-complexity heuristics in Section 5. Simulation results that compare the different algorithms in terms of completion time and energy consumption are presented in Section 6. Finally Section 7 concludes the paper.
2 RELATED WORK

The idea of OMS was pioneered by Gopala and Gamal [1] who studied the tradeoff between multiuser diversity and multicast gain. They analyzed the performance of three different scheduling mechanisms that adapt the transmit rate to the user with the best channel, the worst channel, and the median channel, respectively. In their follow-up paper [7], they investigated the performance achieved by serving a fixed fraction of users. This restriction is relaxed in [12] and [4]. In [12], Ge et al. initiate a transmission if the multicast threshold is satisfied. As the threshold is pre-determined, the transmission rate at each slot is fixed. This limits achievable throughput in case all the receivers have good channel conditions in a slot and higher rate could be used. Kozat et al. show that dynamic selection ratios that select more than 50% of the users can achieve higher throughput [4]. They also present an algorithm where the user selection ratio depends on the channels of the receivers. Both [12] and [4] exploit erasure coding for reliable transmission.

The authors of [8] propose algorithms with a static selection ratio (fixed for all transmissions) and a dynamic selection ratio (adapted to the instantaneous channels for each transmission) that maximize overall throughput. In [2], the authors extend their work of [8] from homogeneous to heterogeneous scenarios, composed of different groups of homogeneous users. A similar optimization algorithm for multicast throughput maximization is proposed in [9]. While all of these works target the infinite horizon case, in [13] the authors consider scenarios with a finite number of multicast packets. Using extreme value theory, they derive the optimal selection ratio for each transmission that minimizes completion time. In contrast to our work, their optimization algorithm does not consider the state of the receivers in terms of the number of received packets. Wang et al. [14] consider both channel and receiver state for the finite horizon problem using a disjoint formulation technique in which, in a slot, the MCS is chosen to serve the user or users with the highest priority, which is determined based on instantaneous throughput and remaining packets. However, the objective of this paper is to improve the fairness between the users rather than optimizing for throughput or completion time.

All of the above papers use a simple outage based channel model, where packet errors are deterministic. Receivers with channel conditions better than some threshold receive the packet and all other receivers lose the packet. In real wireless systems, packet errors are much more random and depend on noise and interference. In our model, we explicitly take the relationship between the channel conditions, the chosen MCS, and the probability of error into account. Also Ho et al. [15] take the probability of error into account in their formulation for maximizing the opportunistic multicasting gain. However, their schemes is a simple instantaneous throughput maximization and is thus not suitable for the finite horizon problem. In summary, all of the above algorithms – except those of [13] and [14] – focus on the infinite-horizon scenario and are sub-optimal for the finite-horizon case we consider in this paper.

The problem of minimizing the overall delay for all users to receive a certain number of packets is studied in [16] through a dynamic programming approach. This work does not consider erasure coding over a larger block of data, but repeatedly multicasts a single packet until each receiver has obtained it. The BS then multicasts the next packet in the same manner, and so on. The goal of the optimization algorithm is therefore to minimize the number of transmissions required to multicast a single packet to all receivers, and the state of the system is the number of receivers that did not yet receive the packet. The algorithm adapts its decision to the changes in the set of users that still need to receive the packet and maximizes the throughput for those users. The approach is mainly suitable for a single homogeneous group of users, since its complexity increases exponentially with the number of user groups in heterogeneous scenarios. The method of multicasting a single packet repeatedly is also much less efficient than multicasting blocks of erasure coded packets as is done above.

The most basic scheme against which we compare our proposed algorithms is the Broadcast algorithm (called LCG user rate in [3]), where the transmission rate is limited by the receiver that currently has the worst channel. This scheme ensures successful transmission to all receivers, but may sacrifice a lot of throughput when channels are highly variable. We further compare against a scheme called Greedy that optimizes the selection ratio at each transmission opportunistically based on the current channel states of all receivers so as to maximize total throughput. This mechanism has a performance that is indicative of the different selection ratio based throughput maximization algorithms above.

Opportunistic multicast algorithm highly depends on the availability of the instantaneous channel information from the users. The impact of limited feedback is examined in [15] and [17]. In [15], the MCS decision is made based on the average SNR. This assumption is too conservative since a realistic channel is also characterized by the path loss, Rayleigh fading and shadowing. Huang et al. [17] determine the transmission rate for the opportunistic multicast scheduling in [13] based on the recent channel conditions instead of the average SNR. Recent channels, however, do not always accurately reflect the instantaneous channel and it highly depends on the feedback interval, which is not explicitly defined.

In summary, our main contribution over prior work is the optimization of completion time for finite horizon multicasting. We further model the packet error rate rather than just considering outage, and evaluate the proposed algorithms under multipath Rayleigh fading. Lastly, we also propose an estimation based algorithm that accounts for imperfect receiver and channel state information.
3 System Model

We model the system as a time-slotted broadcast system with a single Base Station (BS) and \( N \) mobile users within the coverage area of the BS. Each user must receive a block of data of \( B \) bits, called the block size. We assume that due to erasure coding, each packet transmitted by the BS and received by a user is useful if the user has received less than \( B \) bits. In case multiple blocks of data are to be transmitted in succession, the BS will start transmitting the next data block only after all the receivers received the current block.

A time slot is of fixed duration. Thus, the BS broadcasts a fixed number of symbols per slot, which – depending on the MCS – corresponds to a variable number of bits. We assume that the BS can select one of the \( M \) MCSs, indexed by \( m = 1, \ldots, M \). The number of bits per slot that can be transmitted using MCS \( m \) is denoted by \( R_m \).

Perfect Channel State Information (CSI) and knowledge of the number of bits a user has successfully received is assumed to be available at the base station prior to the transmission in each time slot. The users see independent channel instances \( h_i[k] \) at each time slot \( k \). The discrete-time channel model for the received signal \( s_i^R[k] \) at user \( i \) is given by:

\[
s_i^R[k] = h_i[k]s^b[k] + n_i[k],
\]

(1)

where \( s^b[k] \) is the signal broadcast from the BS at time slot \( k \), and \( n_i[k] \) is additive white Gaussian noise term with power spectral density \( N_0 \).

For the analysis, and in particular for dynamic programming solution, we assume a discrete set of \( C \) possible channel realizations \( H_i \) for user \( i \). However, the heuristics we develop also work with continuous channels. The probability of user \( i \) seeing channel coefficient \( h_i \in H_i \) in slot \( k \) is \( \alpha_i(h_i) \), i.e., \( \alpha_i(h_i) = P(h_i[k] = h_i) \). The corresponding SNR is denoted by \( \gamma_i(h_i) \). The vector of channels of all users, also referred to as the channel combination, is denoted by \( h = \{h_i, i = 1 \ldots, N\} \). We denote by \( H \) the set of all possible channel combinations, and by \( \alpha(h) = \prod_{i=1}^{N} \alpha_i(h_i) \), the probability of a channel combination \( h \in H \). Note that the total number of channel combinations is \( C^N \).

In contrast to prior work, we do not assume deterministic channel outage but use the packet error rate (PER) for a given channel quality and MCS from [18]. For a channel instance \( h_i \in H_i \), the PER for user \( i \) under MCS \( m \) is represented by \( p_i^m(h_i) \), and the corresponding packet success rate (PSR) is \( q_i^m(h_i) = 1 - p_i^m(h_i) \).

In this paper, we use the following terms: (1) A strategy \( g \) specifies the MCS \( g(h) \) for each channel combination \( h \in H \). Hence, the total number of strategies is \( S = MC^N \). We denote by \( G \), the set of all possible strategies. (2) The state consists of the vector of the number of bits received by each user \( i \), denoted by \( x = \{x_i, i = 1, \ldots, N\} \). The state space \( \mathcal{X} \) consists of all states where the number of bits received by all users is positive and less than or equal to \( B \). The initial state where none of the users have any information is \( x^0 \) and the end state where all the users have received \( B \) bits is denoted by \( x^B \). (3) A policy \( \mu \) maps any given state \( x \in \mathcal{X} \) to the strategy \( g^\mu_x \) to be used in that state. (4) The expected completion time \( D_\mu(x) \) is the mean time required to get from state \( x \) to the end state \( x^B \) under policy \( \mu \).

4 Optimization Problem

In this section, we consider the case of memoryless channels and formulate the problem as a stochastic shortest path problem [19] with cost per stage equal to 1 (the time needed per slot is fixed, \( \tau = 1 \)) and no terminal cost. We assume that the probability of successfully receiving a packet is non-zero for every combination of MCS and channel condition, though it might be extremely low for some combinations.

4.1 Dynamic programming solution (Dyn-Prog)

Let \( \mathcal{E} = \{e \mid |e| = N, e_i \in \{0,1\} \} \) be the set of all vectors of size \( N \) whose components take values 0 or 1. The transition probability from state \( x \in \mathcal{X} \) to state \( y \in \mathcal{X} \) when MCS \( m \) is used under channel combination \( h \) is given by:

\[
p_i^m(h, x, y) = \min_{e \in \mathcal{E}} \left\{ \sum_{e \in \mathcal{E}} \left( \prod_{i=1}^{N} p_i^m(h_i)^{e_i} q_i^m(h_i)^{1-e_i} \right) \right\},
\]

(2)

where the above minimization is defined element-wise. Note that in case every user experiences an erasure, the state remains unchanged.

The state space is finite, and there clearly exists a finite integer \( K \) such that there is a positive probability of terminating after \( K \) steps irrespective of the policy. Thus, the optimal policy \( \mu^* \) satisfies Bellman’s equations for every state \( x \):

\[
D_{\mu^*}(x) = \min_{g \in G} \left( \tau + \sum_{h \in H} \alpha(h) \left( \sum_{y \in \mathcal{X}} \rho^g(h, x, y) D_{\mu^*}(y) \right) \right),
\]

(3)

and the optimal strategy in state \( x \) is given by:

\[
g^\mu(x) = \arg\min_{g \in G} \left( \sum_{h \in H} \alpha(h) \left( \sum_{y \in \mathcal{X}} \rho^g(h, x, y) D_{\mu^*}(y) \right) \right).
\]

(4)

Since the state space is finite, there are several options to solve for the optimal policy as well as the minimum expected completion time. We choose a simple value iteration approach. Starting from the end state \( x^B \), we use Bellman’s equation Eq. 3 to determine the completion

1. Note that to reduce the state space, we can use a normalized block size that is measured in units of the greatest common divisor of the MCS rates instead of in bits together with a corresponding normalized rate. As an example, for a block size of \( B = 1800 \)kbits, \( M = 2 \) and MCSs with rates of 6Mbps and 9Mbps, the normalized block size is \( B = 600 \) and the normalized rate is \( R_1 = 2 \) and \( R_2 = 3 \), respectively.
times of the states that only depend on the end state (for which the completion time is known to be 0). We then proceed in the same manner to determine the expected completion times of states that only depend on states for which the completion time is already known, until the completion times for all states are computed. This process also yields the optimum policies from Eq. 4.

4.2 A simple two user example

Consider a scenario with two users ($N = 2$), with identically distributed channels. Let $\mathcal{H}_1 = \mathcal{H}_2 = \{L, H\}$, and $\mathcal{H} = \{HH, HL, LH, LL\}$, where $L$ and $H$ denote channels with low and high channel quality, respectively. The base station can choose one of three MCSs in each slot. The probability of packet error when MCS $m$ is used is denoted by $p^m(L)$ and $p^m(H)$ for both users under the low and high channel quality, respectively. A strategy is defined by specifying the MCS to be used for each vector channel in $\mathcal{H}$.

Here, Bellman’s equation at state $\{x_1, x_2\}$ is:

$$
D_{\mu^*}(\{x_1, x_2\}) = \\
\tau + \min_{g \in \mathcal{U}} \sum_{h \in \mathcal{H}} \alpha(h) \left( p^g(h_1)p^g(h_2)D_{\mu^*}(\{x_1, x_2\}) \\
+ p^g(h_1)q^g(h_2)D_{\mu^*}(\{x_1, \min(x_2 + R_{g(h_1)}, B)\}) \\
+ q^g(h_1)p^g(h_2)D_{\mu^*}(\{\min(x_1 + R_{g(h_1)}, B), x_2\}) \\
+ q^g(h_1)q^g(h_2)D_{\mu^*}(\{\min(x_1 + R_{g(h_1)}, B), \min(x_2 + R_{g(h_1)}, B)\}) \right).
$$

We evaluate the optimal policy in a scenario where the $H$ and $L$ channels for the users are $\mathcal{H}_1 = \mathcal{H}_2 = \{5dB, 25dB\}$. The probabilities of $L$ and $H$ are $\alpha_1(L) = \alpha_2(L) = 0.75$ and $\alpha_1(H) = \alpha_2(H) = 0.25$. We choose such a highly variable channel, since it makes it easier to demonstrate the tradeoffs that the algorithm makes in the different regions of the state space. The probability of each channel combination in $\mathcal{H}$ can be easily obtained by multiplying the respective channel probabilities. For simplicity, we use $M = 3$ MCSs with normalized rates of $R_1 = 1$, $R_2 = 4$ and $R_3 = 9$. The PER for each MCS and channel instance is listed in Table 1.

| TABLE 1 |
|------------------|-----------|-----------|-----------|
| $p^g(h_1)$ | $m = 1$ | $m = 2$ | $m = 3$ |
| $\gamma_L = 28dB$ | $0$ | $0$ | $0.08$ |
| $\gamma_L = 5dB$ | $0.23$ | $0.97$ | $1$ |

In Fig. 1, a drift vector (arrow) reflects the optimal policy at that state. It shows the expected future state, given the optimum MCSs chosen for the different channel combinations. Hence, the length of a vector indicates the throughput obtained by the corresponding policy. (Note that for better readability, we only plot policy vectors for a subset of states and increase their lengths.) We set the normalized block size $B = 200$. At the initial state $x^0 = \{0, 0\}$ the optimal policy is $g(h) = \{1, 3, 3, 3\}$, i.e., MCS $m = 1$ is used for channel combination $LL$ and MCS $m = 3$ is used for channel combinations $LH, HL$, and $HH$. This particular policy is a greedy policy which gives the maximum throughput to both users. This policy is also used in almost all states up to $x = \{140, 140\}$. Closer to the boundaries of the state space, the policy changes from greedy to increasingly favoring the user that is lagging behind. This accounts for the fact that the leading user is likely to finish before the trailing user, even if MCS decisions are optimized for the trailing user. It aims to prevent the loss of multiuser diversity caused by one user finishing early. For $\{x_1 < 160, x_2 < 140\}$ and $\{x_1 < 140, 140 < x_2 \leq 160\}$, the predominant policies are $g(h) = \{1, 3, 2, 3\}$ and $g(h) = \{1, 2, 3, 3\}$, respectively, where a more conservative MCS is chosen when the trailing user has a low channel quality. This policy sacrifices throughput to prevent the trailing user from falling further behind. Even closer to the boundaries, the policies are $g(h) = \{1, 3, 1, 3\}$ and $g(h) = \{1, 3, 3, 3\}$, further trading off overall throughput for a higher packet reception probability for the trailing user.

While solving the stochastic shortest path problem minimizes average completion time and provides the optimal policy, the size of the state space and the computational complexity increase exponentially with $N$, the number of users. Therefore, the above approach is not practical for actual implementation in a network.

5 STATE-AWARE HEURISTICS

Due to the high complexity of the Dyn-Prog algorithm introduced in the previous section, we propose two low-complexity heuristics that mimic its behavior.

5.1 Maximize minimum throughput (Max-Min)

This heuristic is based only on the current state and the current channel conditions. At each slot, the user with the least number of received bits is identified as the worst user, who is most likely to require the highest number of slots to receive all data. Note that this is indeed the case when the users are homogeneous and have an identical channel distributions. In the case of heterogeneous users, the users that have channel conditions that are worse (on average) are more likely to be the trailing users and thus most likely to finish last. In each slot, the algorithm uses the MCS that maximizes the throughput for the trailing user. If both users have the same number of bits, the algorithm greedily maximizes sum throughput for both. Note that since the algorithm ignores by how much the worst user is trailing, it may react too conservatively in case the difference between users is small.
Fig. 1. Policy given by the (Dyn-Prog) algorithm

Fig. 2. Policy given by the Max-Min algorithm

Fig. 3. Policy given by the Weighted-CT algorithm

5.2 Weighted completion time (Weighted-CT)

In many cases, favoring the trailing user is overly conservative. In particular, when the number of pending bits is large for all users and the relative lag is small, the probability that the currently trailing user actually finishes last may be small. We now present a heuristic that more closely models the decisions taken by the Dyn-Prog algorithm to achieve a better tradeoff between instantaneous sum throughput and balancing the number of pending bits (i.e., the state) for the different users.

At a given slot $k$, with state $x$ and channel $h$, we evaluate the average drift and determine the expected next state, $y^m$, when using MCS $m$ as:

$$y^m_i = x_i + q^m_i(h_i[k])R_m, \quad i = 1, \ldots, N$$

Note that the expected state may be real valued. We then estimate the additional time required, on average, for all users to receive $B$ bits given that they are in state $y^m$. Since computing the actual estimated remaining completion time is computationally intensive as discussed in the previous section, we use a weighted Euclidean distance metric instead. The distance is taken as the bits required for completion, divided by a weight that reflects the average rate at which a user progresses through the state space. The estimated completion time $\tau_{ym}$ is thus:

$$\tau_{ym} = \frac{\sum_{i=1}^{N} \left( \frac{B - y^m_i}{w_i} \right)^2}{w_i}$$

and the chosen optimum MCS is $m^* = \arg \min_m \tau_{ym}$.

We choose weight $w_i$ that is proportional to the average throughput achieved by the user under a hypothetical policy that chooses the MCS that maximizes the rate of each channel:

$$w_i = \sum_{h_i \in H_i} \max_m \alpha_i(h_i)q^m_i(h_i)R_m.$$ 

As the actual choice of MCSs in the algorithm in turn depends on the policy (and thus on the states as well as the channels of the other users), it is hard to determine the true average rate. At the same time, it is not necessary to estimate this rate very accurately; it is only necessary to obtain approximately the right relative differences in user’s completion time estimates that lead to the correct choice of MCS $m^*$. The hypothetical policy to determine the weights above is a very simple but effective method to capture these approximate relative throughput differences among the users.

In practical scenarios, the channel distribution of individual users may not be known in advance. Further, the channel statistics of a mobile user may change over time. In such settings, we use an exponentially weighted moving average to track the user’s weight. Given an instantaneous channel instance $h_i[k]$ at slot $k$, the estimated weight of user $i$, $\hat{w}_i[k]$, is given by:

$$\hat{w}_i[k] = (1 - \beta)\hat{w}_i[k-1] + \beta \sum_{m=1}^{M} q^m_i(h_i[k])R_m,$$

where $\beta$ is a sufficiently small constant.
Fig. 3 shows the drifts for the Weighted-CT algorithm. Our choice of weights indeed captures well the relative desirability of the different states. While the set of strategies is not as rich as with the Dyn-Prog approach – in particular at the transition between the greedy throughput maximization strategy and the more conservative border strategies – the strategies in most of the state space are almost the same. Most importantly, this is true for states around the diagonal which are much more likely to occur in reality than states far off the diagonal where the number of bits for the two users differs a lot.

5.3 Weighted-CT with rate estimation (Weighted-CTe)

In the previous sections, we assumed that perfect channel and state information is available at the BS. In practice, however, feedback from receivers is delayed and reporting channel and receiver states information incurs overhead and can thus only be done periodically. To deal with such imperfect and outdated information, we design an estimation-based version of the algorithm, Weighted-CTe. It estimates the probability distribution of the current channel state based on the outdated past feedback and predicts the evolution of receiver states.

Assume that $\sigma$ is the delay between the actual channel measurement at user $i$ and the use of that information at the sender. The sender can now estimate the probability of the current channel $h_i[k]$ having the channel state $h_i$, given the outdated channel information $h_i[k-\sigma]$, as

$$\hat{\alpha}_i^k(h_i) = P(h_i[k] = h_i \in H_i | h_i[k-\sigma]).$$

The estimated reception probability of user $i$ when MCS $m$ is used is:

$$\hat{q}_i^m(h_i[k-\sigma]) = \sum_{h_i \in H_i} \hat{\alpha}_i^k(h_i)q_i^m(h_i).$$

With this, Eq. 5, Eq. 7 and Eq. 8 can be rewritten, replacing the actual reception probability for a known channel $q_i^m(h_i[k])$ with the estimated reception probability $\hat{q}_i^m(h_i[k-\sigma])$.

6 Results

In this section, we evaluate the performance of our proposed algorithms in homogeneous and heterogeneous user scenarios and compare them to the existing Broadcast and Greedy schemes discussed in Section 2. For simple scenarios ($N = 2$ and $C = 2$), we also compare our results to the optimal Dyn-Prog solution. We start with simple scenarios to provide an intuition for the algorithms that helps to better understand the more complex scenarios. We then study the impact of block size $B$ and the number of users $N$ on the performance of the algorithms under multipath Rayleigh fading channels with the ITU Pedestrian B path loss model [20]. The Doppler frequency is 10 Hz and the coherence time is $t_c = 40$ ms. Given that multicast traffic will be sent concurrently with other unicast data traffic, only some of the slots of an LTE frame can be used for multicast. In the simulations with Rayleigh fading, we use two out of the ten slots (or sub-frames) of a frame for multicast. Each slot has a duration of 1 ms and thus the transmission time interval (TTI) is equivalent to 5 ms. Finally, we analyze the performance in a more practical scenario with limited and imperfect feedback from the users.

The algorithms are evaluated with two performance metrics: the system completion time $D$ (in all scenarios) and the energy consumption (in the multipath Rayleigh fading scenarios). The completion time $D$ corresponds to the time required for all of the users in the system to receive the whole block of data. It is measured in slots of $1$ ms. The energy consumption is measured in Joule.

In all of the simulations, we consider three modulation schemes: QPSK, 16-QAM, and 64-QAM, with channel coding and data rates that are corresponding to CQI=3 to CQI=15 in [18]. The MCS determines the number of transmitted bits and the PER for the instantaneous channel quality in a slot. The block size $B$ used throughout this section is 6400kbits unless otherwise specified. This block size roughly corresponds to the size of a group of pictures (GOP) for a video with DVD quality [21].

6.1 Completion time comparison to the optimal Dyn-Prog solution in simple scenarios

In this section, we first analyze the performance of the different algorithms in a simple $N = 2$ user scenario with $C = 2$ channel instances and $M = 13$ MCSs (CQI = 3 to CQI = 15 in [18]). In such a simple scenario, it is possible to obtain the optimum Dyn-Prog solution. The system model that we use here is the one described in Section 4.2. We analyze both homogeneous and heterogeneous user scenarios. In these scenarios, we set the stationary channel probabilities for the $H$ and $L$ channel to $\alpha(H) = 0.25$ and $\alpha(L) = 0.75$, respectively.

6.1.1 Homogeneous network

In the homogeneous scenario, both users have the same channel statistics but independent channel instances. We present the results of increasing the channel variability $\delta$, where $\delta$ is the SNR difference between the $H$ and $L$ channel of each user. Here, the lowest $\delta$ is 0.7dB ($\gamma_H = 9.0$dB and $\gamma_L = 8.3$dB) and the highest $\delta$ is 20.0dB ($\gamma_H = 21.0$dB and $\gamma_L = 1.0$dB). The $H$ and $L$ channel pair of a user is picked such that the average throughput the user would obtain in a single-user scenario does not change. As $\delta$ increases, the SNR of the $H$ channel increases and the SNR of the $L$ channel decreases.

Fig. 4 shows how channel variation impacts the performance of the algorithms in a homogeneous scenario. In this scenario, both Greedy and Weighted-CT perform close to the optimal Dyn-Prog solution. As the users have the same channel distribution, exploiting opportunistic gain and maximizing the instantaneous throughput as Greedy does is a good strategy. The low complexity
heuristic, Weighted-CT, weighs the homogeneous users equally according to Eq. 7 and transmits with the MCS that leads to the expected future state with the lowest completion time using Eq. 5 and Eq. 6. Weighted-CT trades off the instantaneous opportunistic throughput and the homogeneity of the receiver states to minimize the completion time in a manner similar to Dyn-Prog. It thus performs slightly closer to Dyn-Prog than Greedy. In contrast to Greedy, Weighted-CT exploits opportunistic multicast less aggressively in case this leads to one of the users trailing too far behind. However, since the users are homogeneous, this does not happen often and therefore the performance differences are small.

Broadcast performs worse than the other schemes because its transmission rate is limited by the lowest instantaneous channel. The impact is more pronounced when $\delta$ is larger since the SNR of the $L$ channel is lower. Generally, Max-Min performs worse than the other schemes but Broadcast. In case the trailing user has a better instantaneous channel than the other user, Max-Min transmits at a higher rate than Broadcast, which is beneficial in a homogeneous scenario since exploiting opportunistic gain is a good strategy. However, when the trailing user has a worse instantaneous channel, Max-Min may send at the broadcast rate, and thus performs worse than Dyn-Prog, Weighted-CT, and Greedy. For $\delta = 3.7$ dB, transmitting at the broadcast rate is the right decision. Here, all schemes transmit at the broadcast rate except for Max-Min, which thus performs worse.

Note that the completion time of Greedy, Weighted-CT and Dyn-Prog is not constant – it first increases, then slightly decreases – although the hypothetical single user throughput would be equal for the different $\delta$, as described in the setup above. This change is due to the fact that the algorithms select the transmit rate depending on the channel instances of both users and the distribution of transmit rates is different from the single-user case.²

² For example, in the single-user case, $\mathcal{H} = \{H, L\}$ and the probability to transmit at the rate that corresponds to $H$ is $\alpha(H) = 0.25$ and $\alpha(L) = 0.75$ for $L$. In the two-user case, $\mathcal{H} = \{HH,HL,LH,LL\}$. Here, maximizing the opportunistic gain is optimal and the probability to transmit at the rate for $H$ is $\alpha(H) = \alpha(HH) + \alpha(HL) + \alpha(LH) = 0.44$ and at the one for $L$ is $\alpha(L) = \alpha(LL) = 0.56$.

6.1.2 Heterogeneous network

In a heterogeneous channel scenario, the good user ($g$) has a better average channel than the bad user ($b$). In Fig. 5, we evaluate the completion time when increasing the average SNR of the good user $\gamma_g$ from 2.8 dB to 22.8 dB and fixing the average SNR of the bad user $\gamma_b$ at 2.8 dB. Higher average SNR results in a higher rate, thus as an overall trend the completion time decreases. Here, the difference between each of the $H$ and $L$ channels of both users is always $\delta = 12.4$ dB.

On the left extreme of Fig. 5 (at $\gamma_g = 2.8$ dB), the users are homogeneous and the relative performance of the algorithms is as discussed in Section 6.1.1. Weighted-CT performs close to the optimal Dyn-Prog for all $\gamma_g$. This confirms that also for heterogeneous scenarios, the computation of $y^{m}_{i}$ and $\gamma^{m}_{y}$ based on the average rate estimate $w_i$ leads to MCS decisions almost identical to those of Dyn-Prog. When $\gamma_g$ is low (users are homogeneous), it maximizes aggregate user throughput, whereas when $\gamma_g$ is high (users are heterogeneous), it is more conservative towards the bad user.

When the difference between the good and the bad user is sufficiently large (at $\gamma_g = 8.8$ dB), Greedy’s completion time increases drastically and for higher $\gamma_g$ it performs worse than the other algorithms. For these high channel differences, serving the good user alone provides a higher sum throughput than serving both users. Therefore Greedy serves users sequentially – first the good user at a high rate until the user finishes and only then the bad user. In contrast, Broadcast and Max-Min schemes which always favor the trailing user outperform the Greedy scheme. Broadcast is the best strategy when it is optimal to only serve the user with the worst channel (for $\gamma_g \geq 14.8$ dB). At $\gamma_g = 12.8$ dB, Max-Min performs slightly worse than Broadcast because with some small probability the good user may still trail, causing the algorithm to transmit at a too high rate resulting in a very low PSR at the bad user. Max-Min experiences an increase in $D$ for $\gamma_g = 4.8$ dB for the same reason.

To further analyze the algorithms, we show the instantaneous sum throughput at each time slot, averaged over 200 simulation runs. Instantaneous sum throughput is the total throughput of the users remaining in the system.
in a given slot. Fig. 6 shows a homogeneous scenario, corresponding to $\gamma^g = 2.8$dB in Fig. 5. Fig. 7 shows a heterogeneous scenario, corresponding to $\gamma^g = 12.8$dB in Fig. 5. Note that the user throughput is zero for a user that has already received all the data.

It is optimal to exploit opportunistic gain and serve the better user when the users are homogeneous. Therefore, Fig. 6 shows that Broadcast achieves a lower sum throughput than the other schemes because its rate is limited by the worst instantaneous channel. Consequently, the users finish later than the other schemes (taking between 500 to 600 slots). Max-Min transmits at a higher rate when the trailing user has a better channel and thus achieves higher sum throughput than Broadcast. As discussed before, the performance of Weighted-CT and Greedy is close to the Dyn-Prog scheme, which is also evident in the sum throughput curves.

Fig. 7 shows the sum throughput of the schemes when users are heterogeneous. At time slots $\leq 100$, Greedy has the highest sum throughput because it first transmits at a high rate to the good user only, to maximize opportunistic gain. The good user can be served at around three times the rate of the bad user. Consequently the sum throughput drops significantly in time slot $\approx 100$ when the good user finishes, leaving only the bad user in the system. Clearly, in this scenario serving primarily the bad user is the better strategy, since also the good user will receive those data.

In contrast to Greedy, with Dyn-Prog, Broadcast, and Weighted-CT the users finish relatively close together, since the algorithms serve both users simultaneously. However, Dyn-Prog has a higher sum throughput than Max-Min, Broadcast, and Weighted-CT for time slots $\leq 100$. The optimum strategy is to opportunistically favor the good user as long as both users have a small difference in terms of the amount of received data. This effect is depicted in Fig. 8 where the state space visited by the Dyn-Prog algorithm is shown. The darker the square, the more often the corresponding states are visited. The good user receives more data than the bad user at the beginning of the transmission (i.e., when the bad user has 2000kbits, the good user usually has $\approx 3000$kbits, but may have as few as 2000kbits). For time slots $\geq 100$ in Fig. 7, Dyn-Prog transmits at the rate of the trailing bad user to ensure that the bad user catches up. Since a scheme that transmits at the rate of the bad user sometimes serves only the bad user, Dyn-Prog yields a lower sum throughput than Broadcast, Max-Min, and Weighted-CT for time slots $\geq 100$. Fig. 8 reflects the characteristic of Dyn-Prog serving the bad user for $x_{bad} \geq 1500$kbits and $x_{good} \geq 2500$kbits where the progress along the x-axis (bad user) is larger than that for the good user.

Weighted-CT has a lower sum throughput than Dyn-Prog (see Fig. 7) for time slots $\leq 100$ because the computation of the expected completion time is sub-optimal – it does not take into account all of the (exponential number of) strategies that the optimal Dyn-Prog algorithm explores. From Fig. 9, we see that the sub-optimality of Weighted-CT causes it to be more conservative at the beginning of the transmission, and in turn achieves a slightly higher rate later on since the states of the good and bad user are closer together. Nonetheless, Weighted-CT performs very close to Dyn-Prog compared to the other algorithms for all $\gamma^g$ since Dyn-Prog’s slightly more aggressive initial behavior only provides a marginal reduction in completion time.

In the following, we use the flat multipath Rayleigh fading model with path loss (ITU-R Pedestrian B [20]) and evaluate the performance in both homogeneous and heterogeneous scenarios for different numbers of users $N$ (see Section 6.2.1) and block sizes $B$ (see Section 6.2.2). From this section onwards, due to the complexity of Dyn-Prog mentioned in Section 5, we only evaluate the performance of Weighted-CT, Greedy, Max-Min and Broadcast.

In a homogeneous scenario, we place the users equidistant from the BS while in the heterogeneous scenario, the users are randomly distributed within the cell coverage area of radius 250m. The multipath Rayleigh channel distributions of the users are independent and identically distributed (i.i.d.). The transmit power is set such that the edge user is still able to receive data with some probability of success at the lowest MCS. Note that since we use the pedestrian channel model, the rate of change in channel SNR is rather low.
6.2.1 Impact of increasing the number of users $N$

Here, we observe the impact of increasing $N$ exponentially from 2 to 64 and fixing $B$ at 6400 kbits. Fig. 10 and Fig. 11 show this impact for a homogeneous and heterogeneous scenario, respectively. The completion time increases with $N$ because a higher $N$ increases the probability that there is a user with a low SNR channel. To make it easier to compare the relative performance of the schemes, we also include a graph that shows the relative increase of the completion time for each scheme compared to the best scheme, for each scenario.

**Homogeneous scenario.** The channel variation (i.e., the difference between the best and the worst channel instance) of the Rayleigh fading channel is high and thus we can observe similar relative performance between the schemes in Fig. 10 and Fig. 4 (a homogeneous 2 user scenario in Section 6.1.1) for $\delta \geq 10$dB. As mentioned, the optimal scheme for homogeneous scenario is the one that exploits opportunism and transmits at a higher rate to achieve maximum throughput at each time instant. Therefore Weighted-CT and Greedy achieve a lower $D$ compared to the other schemes. As $N$ increases, which user is the trailing user changes more often and the trailing user may be a user with a better channel. Since it is better to opportunistically serve users with better channel in a homogeneous scenario, Max-Min performs better than Broadcast. Broadcast performs worst and its completion time increases with $N$ because the probability that there exist a user with a very low SNR in a given slot is higher for higher $N$.

**Heterogeneous scenario.** In Fig. 11, the heterogeneity (i.e., the difference of the average channels) between a user and its nearest neighbor is higher when the number of users in the system is small. Therefore, we observe that the relative performance of the schemes in a Rayleigh fading channel (see Fig. 11) for small $N$ is similar to the performance of the heterogeneous 2 user scenario (see Fig. 5 in Section 6.1.2) for higher $\bar{\gamma}$. According to the explanation in Section 6.1.2, Broadcast performs close to optimal and Greedy that optimizes for opportunistic gain performs worst when there is one clearly worst user. As $N$ increases, the user’s density increases and thus the users that are close to each other have a very small difference in terms of the average channels. This resembles a homogeneous scenario. In such a scenario, transmitting at the broadcast rate and conservatively serving all users results in higher $D$ than using opportunistic gain among the users that are near each other. Therefore Greedy performs better than Broadcast for higher $N$. For high $N$, the multicast rate is highly affected by the users located at the edge of a cell (i.e., edge users). Since the trailing user is normally an edge user, Max-Min may by chance serve the correct users. Therefore, it outperforms Greedy for serving the important users and Broadcast since it is less conservative. As before, Weighted-CT outperforms all other schemes since it exploits the users’ weight and optimizes the rates at which the edge users are served.

6.2.2 Impact of increasing the block size $B$

Here, we fix $N$ to 16 and examine the impact of increasing $B$ exponentially from 50 kbits to 25600 kbits. For ease of comparison, the results are presented in terms of normalized completion time, $D_n = D/B$. Fig. 12 and
6.3 Evaluation of the Energy Consumption

This section presents the energy consumption of the experimented schemes in the homogeneous and heterogeneous multipath Rayleigh fading scenarios with perfect feedback.

At slot $k$, a user is either in the on state or in the idle state. A user is in the on state if it is scheduled for reception and idle state otherwise. The parameters for computing the energy consumption are listed in Table 2.

![Energy Consumption Diagram](image)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi_{on}$</td>
<td>on’s state base power</td>
<td>$1210.7 \pm 85.6$ (mW)</td>
</tr>
<tr>
<td>$\xi_{idle}$</td>
<td>idle’s state base power</td>
<td>$394.3 \pm 8.7$ (mW)</td>
</tr>
<tr>
<td>$\theta_{dl}$</td>
<td>Power per Mbps</td>
<td>$81.97$ (mW/Mbps)</td>
</tr>
</tbody>
</table>

As expected, the energy consumption is proportional to the completion time. This can be seen in the homogeneous scenario where a scheme with the lowest completion time (Weighted-CT) in Fig. 10 (see Section 6.2.1) has the lowest energy consumption in Fig. 14.

Although Weighted-CT has lower completion time than Greedy in Fig. 11, its average energy consumption is higher (see Fig. 15). The energy consumption is not only affected by the completion time but it is also affected by the number of slots a user stays in the on state. Table 2 shows that a user in the on state consumes more than twice the amount of base energy than it does in the idle state. In a heterogeneous scenario, Weighted-CT consumes higher average energy than Greedy because it tries to serve more users and transmits at a lower rate than Greedy. Transmitting at lower rate causes the better users to stay in the on state for more slots. In contrast, Greedy first transmits at a high rate and reduces the number of slots the better users stay in the on state. As a result, its average energy consumption is lower. Increasing $N$ does not decrease the rates at which users are served hence the average energy consumption of

3. The energy consumption in LTE system is presented in [22] and [23]. We ignore the energy consumed by the wake up operation since it is less than 5% of the energy consume in an idle state [22].
Greedy remains the same as $N$ increases. In contrast, for Weighted-CT, a higher $N$ causes more users to stay in the system for a longer time and thus its average energy consumption increases with $N$ in Fig. 15.

In summary, the average energy consumption depends on two main factors: the completion time and the percentage of the slots the users stay in the system (especially in the on state).

### 6.4 Impact of imperfect and limited state information

The schemes presented in this paper need either the channel state or the receiver state information or both to select an MCS. In wireless networks, frequent feedback of the state information substantially increases system overhead and thus reduces system throughput. In this section, we examine the impact of imperfect feedback information on all schemes. We also include Weighted-CTe that estimates the probability distribution of the current channel state based on the outdated past feedback (see Section 5.3). First, we analyze the impact of increasing the feedback interval (i.e., reducing the frequency of feedback) of the state information. We then study the effect of limiting the number of users that periodically feed back their state information to the BS.

#### 6.4.1 Impact of the feedback interval

As mentioned above, transmitting feedback at each slot is costly. Here, we model a feedback system where users only send feedback every $\lambda$ slots. We then analyze the impact of the feedback interval $\lambda$ on the completion time $D$. The feedback slot (the slot at which a user reports) is asynchronous, but $\lambda$ is the same for all users.

We choose $\lambda \in \{5\text{ms}, 10\text{ms}, 20\text{ms}, 40\text{ms}, 80\text{ms}, 160\text{ms}\}$ (these are the common periodic feedback intervals in an LTE system [24]), such that we can observe the impact of $\lambda$ when it is less than or greater than $t_c = 40\text{ms}$. For $\lambda \leq t_c$, the current channel state and the one reported in the last feedback (i.e., the last available channel state) are correlated, otherwise, they are uncorrelated. When feedback is unavailable, the BS updates the receiver state assuming that the amount of data received in the current slot is equivalent to that reported in the last available feedback packet. When feedback is available, the BS updates the receiver state to the current receiver state.

Fig. 16 and Fig. 17 depict the impact of different feedback intervals $\lambda$ on $D$ in the homogeneous and the heterogeneous scenarios, respectively. $D$ increases with $\lambda$ because the correlation between the current channel state and the last available channel state decreases and thus the uncertainty about the channel state increases. As mentioned, the receiver state is updated regardless of the availability of feedback information. The difference between the current and the estimated receiver state is usually small and thus has a minimal impact on the chosen MCS. Hence, delayed channel state has a much higher impact than the receiver state on the completion time.

![Fig. 16. Impact of increasing $\lambda$ for a homogeneous multipath Rayleigh fading scenario, $B = 6400 \text{kbits}$, $N = 16$.](image1)

![Fig. 17. Impact of increasing $\lambda$ for a heterogeneous multipath Rayleigh fading scenario, $B = 6400 \text{kbits}$, $N = 16$.](image2)

Fig. 16 and Fig. 17 show that $D$ continuously increases with $\lambda$. For $\lambda > t_c$, the last available channel state is only useful up to time $t_c$. From time $t_c$ until the next feedback is received, the current and the last available channel state are uncorrelated. Therefore, increasing $\lambda$ beyond $t_c$ increases the fraction of time where the current and the last available channel state are uncorrelated. When comparing the impact of $\lambda$ on $D$ in the homogeneous (see Fig. 16) and the heterogeneous (see Fig. 17) scenarios, the impact is greater in the latter. In a homogeneous scenario (with a sufficiently large $N$), the difference between the chosen MCS based on the last available channel state and the one that would be chosen based on the current channel state is usually small. In contrast, in a heterogeneous scenario, an outdated channel state may cause an algorithm to select a very different MCS compared to the MCS that would be selected based on the current channel state since the MCS largely depends on the channel state of a small subset of users. Choosing either a too low or too high $\lambda$ increases $D$. As a result, the impact of delayed state information is higher in the heterogeneous scenario than in the homogeneous scenario. For $\lambda > t_c$, although Weighted-CT may make the wrong MCS choice due to outdated last feedback, it performs as good as or better than the other schemes except for Weighted-CTe.

Weighted-CTe outperforms the other schemes in all scenarios, especially for higher $\lambda$. The MCS that is chosen for an outdated channel is the one that is optimal for the distribution of the estimated channel (since little is known about the channel). As a result, Weighted-CTe that picks the MCS using channel estimation significantly outperforms all the other schemes especially for high $\lambda$. With channel estimation, the completion time of Weighted-CTe improves over Weighted-CT by 17.5% and 30% in homogeneous and heterogeneous scenarios, respectively. As mentioned, the difference of the selected MCS between the current channel state and the last available channel state is larger in the heterogeneous scenario therefore the impact of channel estimation is more evident in this scenario.

We also investigate the performance of Broadcast, Max-
Min, and Greedy when channel estimation similar to Weighted-CTe is applied. Due to space limitation, we exclude the graphs. With channel estimation, Max-Min and Greedy achieve improvements of up to 29% and 21%, respectively in terms of $D$ compared to Max-Min and Greedy without channel estimation. In contrast, Broadcast with channel estimation performs worse than all the other schemes, including the Broadcast scheme without channel estimation. This is because the estimated channel of Broadcast is the worst channel in the channel distribution.

6.4.2 Impact of interval feedback from fewer users

In this sub-section, the feedback load is further decreased by reducing the number of reporting users ($N_{rep}$). We examine the minimum $N_{rep}$ for each scheme that achieves the same $D$ as that when all users give feedback. We first elaborate the method on the selection of the reporting users. We then present the minimum $N_{rep}$ for $N = \{2, 4, 8, 16, 32\}$ for a feedback interval $\lambda = 20$ms.

As depicted in Fig. 18 and Fig. 19, $N_{rep}$ increases with $N$ for Weighted-CTe, Weighted-CT, and Greedy because more users are needed to reflect the channel statistics of the important users for each scheme. For Greedy to realize the opportunistic gain, the feedback from one user is as important as any other users in the system and thus a higher $N_{rep}$ is required.

In Fig. 18, it is interesting to note that the minimum $N_{rep}$ of Weighted-CTe is lower than that of Weighted-CT in the homogeneous scenario. As explained in the previous section (Section 6.4.1), a high $\lambda$ reduces the number of MCS choices when channel estimation is used. Therefore, only a small $N_{rep}$ is needed for Weighted-CTe. On the other hand, Weighted-CT has a wider range of MCS choices thus including more users results in smaller $D$. As discussed in Section 6.4.1, in the heterogeneous scenario, it is important to select the edge users since the completion time is determined by those users. The number of the edge users increases with $N$ and thus $N_{rep}$ is larger for larger $N$. Weighted-CT and Weighted-CTe have the same $N_{rep}$ in Fig. 19 except for $N = 16$. This small difference is caused by the number of MCS choices as explained above.

To recap, the minimum number of reporting users $N_{rep}$ depends on the network scenario. Since users are usually randomly distributed in actual mobile networks, they are heterogeneous. As shown in our result (see Fig. 19), the BS (for any algorithm it uses) can achieve its best performance with low overhead because it only needs feedback from a small fraction of users in the system to achieve the minimum completion time for each scheme.

7 Conclusions

In this paper, we investigated the finite horizon opportunistic multicast scheduling problem, where a wireless base station transmits a fixed amount of erasure coded data to a set of receivers. We designed an algorithm based on dynamic programming that to the best of our knowledge, is the first to explicitly take into account the system state in terms of the received amount of data at each receiver for the selection of the optimum modulation and coding scheme. In addition to the well known tradeoff between broadcast gain and multi-user diversity gain that is inherent to opportunistic multicast scheduling, the finite horizon nature of our problem introduces an interesting further tradeoff, namely that of equalizing the completion times of the users versus the total system throughput. This tradeoff is state dependent. Intuitively, throughput maximization is a reasonable strategy as long as users are far from finishing, whereas the closer users are to finishing, the more important it becomes to allow lagging users to catch up rather than optimizing throughput for all. Based on these insights, we designed two simple and practical heuristics that perform close to the more complex dynamic programming solution and that outperform existing approaches that do not consider the receiver state.
We performed an extensive range of simulations for homogeneous as well as heterogeneous user scenarios and show that our heuristics outperform the existing Greedy and Broadcast schemes by as much as 30% and 120%, respectively, in Rayleigh fading scenarios. Since reducing the system overhead improves the system throughput. We also present results on the impact of increasing the feedback interval and reducing the number of users that give feedback on the proposed schemes. In particular, we extend our heuristic to estimate the current channel state based on outdated last feedback. This extension further improves performance by 17.5% and 30% in homogeneous and heterogeneous scenarios, respectively.

References

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